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THE MATHEMATICAL GAZETTE

EDITED FOR THE MATHEMATICAL ASSOCIATION BY
T. A. A. BROADBENT, M.A.
ROYAL NAVAL COLLEGE, GREENWICH, LONDON, S.E. 10.

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SYDNEY CHAPMAN, M.A., D.Sc., F.R.S.
PRESIDENT, APRIL 1945–APRIL 1946.

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VOL. XXX. MAY, 1946. No. 289

THE MATHEMATICAL ASSOCIATION.

A GENERAL MEETING of the Mathematical Association was held at the Polytechnic, Regent Street, on 24th and 25th April, 1946.

On 24th April, the business meeting was held at 10.30 a.m., the President, Professor S. Chapman, in the chair. The Report of the Council for 1945* was adopted. On the nomination of the Council, Mr. W. F. Bushell was elected President for the year 1946. The existing Vice-Presidents were re-elected, and Professor S. Chapman, Professor W. V. D. Hodge and Mr. W. L. Ferrar were elected Vice-Presidents. The Treasurer, the Librarian, the Editor of the *Gazette*, and the Auditors were re-elected. Mr. G. L. Parsons, and Miss M. E. Bowman were elected as Secretaries. The following were elected to serve on the Council: Professor G. B. Jeffery, Mr. F. W. Kellaway, Mr. W. J. Langford, Dr. E. A. Maxwell, Mr. A. W. Riley, Mrs. E. Shuttleworth, Mr. F. J. Swan, Mrs. E. M. Williams.

The revised regulations for the Teaching Committee and the composition of the new Committee were announced.

At 11 a.m. the President delivered his Presidential address, "University Training of Mathematicians".† At 2 p.m. a discussion on "Geometrical drawing in the H.S.C. course in schools" ‡ was opened by Dr. W. G. Bickley and Dr. S. Weikersheimer. At 4.45 p.m. Mr. D. N. de G. Allen read a paper on "Relaxation methods in engineering".‡

On 25th April, at 10 a.m., Mr. D. J. Finney read a paper on "Statistical science and agricultural research" ‡, followed at 11 a.m. by a paper on "A new approach, by means of isogonal conjugates, to certain geometrical theorems and to a general theory of conics", by Mr. H. E. Piggott and Dr. Alfred Steiner.‡ At 2 p.m., a discussion on "The place of mathematics in secondary (modern) schools" ‡ was opened by Mr. F. J. Swan, Mr. C. T. Lear Caton and Mr. H. H. Watts.

At 5 p.m., Professor T. A. A. Broadbent read a paper on "The history and aims of the *Mathematical Gazette*".

A Publishers' Exhibition was open during the two days.

* See pp. 58–60. † See pp. 61–70. ‡ To be published later.

REPORT OF THE COUNCIL FOR THE YEAR 1945.

Membership.

During the year 1945, 131 new members have been admitted, of whom 15 are junior members. The membership now stands at 1772, of whom 7 are honorary members, 144 life members, 1596 ordinary members, and 25 are junior members. The Council has pleasure in welcoming back a large number of members who have been serving in the Forces, including some who, through circumstances outside their own control, were forced to suspend their membership.

Mr. R. J. A. Barnard, one of the founders in 1905 of the Mathematical Association of Victoria (subsequently the Victoria Branch of the M.A.), died on 7th October, 1945, at the age of 78. He was Professor of Mathematics at the Duntroon Military College until it was closed in 1922: thereafter, Senior Lecturer in Mathematics at the University of Melbourne until he reached retirement age. He was the writer of a number of well-known textbooks.

Mr. C. W. Parkes, who died in January 1945, has been a member of the Council and has also served on various Committees of the Association. He served for many years as H.M.I. in Mathematics.

The Council also reports with regret the death of the following members: Mr. F. Robbins, Mr. H. J. Woodall, Dr. J. W. Jenkins, Mr. J. O. Mahoney, Mr. E. H. Smart.

Finance.

Receipts and expenditure have both increased this year to nearly pre-war level, thus reflecting the increased membership and activity of the Association. During the war the grant paid to the Editor of the *Mathematical Gazette* has not come under periodical review as in normal times. Hence the Council have this year made an extra grant of £200 to cover the war years, with the increased cost of living, and to show recognition of the fact that the regular issue of the *Gazette*, under difficult conditions, has been of extreme value to the Association. The annual grant has now been increased to £200. In the next few years there must be greatly increased expenditure in the production of reports, list of members, etc. Although £1000 has been saved during the last six years for this purpose, it is important, from the financial side as well as on general grounds, that steady increase in membership should be maintained.

The Branches.

The Council is making an attempt to revive the work of the Branches Committee, and a meeting is being held in April. The Council considers that the time is opportune to co-ordinate the work of the Branches, which can play a valuable part in the activities of the Association.

The year has seen a further resumption of branch meetings, and all the branches except three have recommenced their activities; the remaining three hope to reopen their meetings this spring. Detailed reports are still lacking in some cases, but it is clear that considerable interest is being shown in the content of the mathematics syllabus, particularly for the first two years of the secondary school period. The Manchester Branch has drawn up a short report on this.

The London Branch has had good meetings, including one devoted to the teaching of Mechanics to the ordinary pupil. In the near future an exhibition of mathematical teaching films is to be arranged. Mr. C. T. L. Caton's address to the Midland Branch was reprinted in the *Gazette*, and Mr. W. W. Sawyer has spoken on the value of History in the teaching of Mathematics.

The Yorkshire, North-Eastern, Sheffield and Plymouth Branches are continuing their successful meetings. The Council is pleased to report a resumption of activity in the Cardiff, Bristol, Southampton and Liverpool Branches. At Southampton Dr. Maunsell, on his return from service, has spoken on the various uses of mathematics encountered during his war experiences. This topic has also appeared at Bristol, and the Bristol Branch has discussed recent trends in examination syllabuses.

Of the Junior Branches, the newest at Birmingham has shown considerable activity under the guidance of Dr. Pedoe. The branch at King's College, London, is also holding meetings, but there is no news so far of the branches at Oxford and Cambridge.

All the overseas branches are very active. At Sydney there has also been discussion of new topics for the school syllabus. The publication of the *Australian Mathematics Teacher*, mentioned elsewhere, is clear evidence that the branch is very much alive. The Queensland Branch has continued its meetings, and meetings of the Victoria Branch have been resumed. Congratulations are due to Mr. F. J. D. Syer, Secretary of this branch, on the award of the M.B.E. whilst on service with the R.A.A.F.

The Mathematical Gazette.

Volume XXIX was slightly larger than Volume XXVIII because of a small increase in the allocation of paper. Difficulties of production and distribution still remain, but it is hoped that in Volume XXX the considerable time-lag between the acceptance of a contribution and its appearance in print may be somewhat reduced.

The task of re-establishing exchanges after the war-time break is now being undertaken.

The Teaching Committee.

The Teaching Committee met in September to consider a first draft of a report on the teaching of Elementary Trigonometry, and it was decided that the first part of that report, which is to be published separately, required further revision. Since that time, progress has been made by the sub-committee concerned, both with the first part and with the rest of the report. The sales of other reports have been such that reprints have been ordered for the Arithmetic, First and Second Geometry, and Mechanics.

Representatives have been nominated to attend conferences on Educational Films and Visual Material, and others to attend a meeting about Standard Symbols to be recommended for use in Mechanics in connexion with Physics teaching.

It will probably fall to the new Teaching Committee to consider in detail the curriculum of Preparatory Schools, since the Headmasters' Conference has recently asked for the comments of the Association on the mathematical parts of their pamphlet relating to these schools.

The Problem Bureau.

The work of the Bureau has continued steadily. During the past year over 150 solutions have been provided, and the members who received them speak with gratitude of the assistance given them from the Bureau.

The majority of questions come from University entrance scholarship papers, including those at Girton and Newnham. From the point of view of subject-matter, Calculus and Mechanics are still in the lead.

Officers and Council.

The Council records its thanks to Prof. S. Chapman for his services as President, and nominates Mr. W. F. Bushell, M.A., F.R.A.S., as President for the ensuing year.

The Council, desiring to strengthen the ties of the Association with the Universities, has decided to make some additional nominations to the office of Vice-President. In addition to those who previously held the office, Prof. S. Chapman, Prof. W. V. D. Hodge and Mr. W. L. Ferrar have been nominated.

In consequence of her appointment as Principal of the Leicester Training College, Mrs. E. M. Williams has felt compelled to relinquish the office of Secretary, which she has held with distinction since 1938. Whilst naturally regretting this decision, the Council offers its sincere thanks to Mrs. Williams for the able and untiring manner in which she has served the Association in difficult times, and also good wishes for her successful tenure of the important post she now holds. Miss M. E. Bowman has been nominated to fill the vacancy.

Miss W. M. Lehfeldt and Mr. A. P. Rollett have resigned from the Council, and the Council wishes to thank them for past services to the Association.

The Council has recently revised the regulations for the Teaching Committee. A new Committee appointed by the Council in January 1946 takes office in April. The Council wishes to thank the members of the retiring Committee, especially the Chairman and Secretary, for devoted work in exceptionally difficult conditions.

The thanks of the Council are again due to Prof. Broadbent for his continued service as Editor of the *Mathematical Gazette*, to Prof. Neville for his work in connection with the Library, to Mr. Gosset Tanner and his keen body of assistants in the Problem Bureau ; as also to the Treasurer, the Secretaries and to the assistants at Gordon Square.

GLEANINGS FAR AND NEAR

1482. *Products of addition.*—" This year of ours 1645, is from the year of the world 7154, which doth exactly agree unto this last account 5509, for if unto that summe be added 1645, the product will be 7154."—*Pseudodoxia Epidemica*, book VI, ch. 1 (1646).

" Here the gentleman has proved by the dint of arithmetic that I contradict myself : for, by adding those 80 years to Ol. xxxviii. 2, the product is Ol. lviii. 2."—*Dissertation upon Phalaris*, ed. of 1699, p. 29.

No one (unless he be a scholar of Christ Church) is to accuse Browne or Bentley of misuse of a word ; " product " has here its general meaning, no deference being paid to the mathematicians' claim to restrict its use in arithmetic to the outcome of the one process of multiplication. But two questions are evoked. Was this general use in an arithmetical context common and persistent? Is it a coincidence that in both instances quoted the context is a reckoning in chronology? [Per Prof. E. H. Neville.]

1483. In most animals (many fishes among the vertebrates and most of the invertebrates except the insects) growth continues at a decreasing rate throughout the adult period. In such animals the curve of growth is, in fact, parabolic ; the growth-rate continuously approaches a zero value but never reaches it.—G. S. Carter, *A General Zoology of the Invertebrates*, 1940, p. 201. [Per Mr. S. Melmore.]

1484. The insects seem to have evolved by the differentiation of a large number of parallel radiating lines which are represented to-day by the insect orders.—G. S. Carter, *A General Zoology of the Invertebrates*, 1940, p. 472. [Per Mr. S. Melmore.]

UNIVERSITY TRAINING OF MATHEMATICIANS.*

By S. Chapman.†

Presidential Address to the Mathematical Association, April, 1946.

The Mediaeval Period.‡

Universities have taught in England for about seven centuries, and mathematics has always been one of the subjects taught; but in the earlier centuries it was not part of the undergraduate course. This consisted of three years' study of Latin grammar, logic, and rhetoric, forming what was called the *trivium*. Mathematics was part of the M.A. course, consisting of arithmetic, geometry, music and astronomy, forming the *quadrivium*; but this post-graduate status meant less than might seem, because the undergraduates had the position of schoolboys today, and became Bachelors of Arts at ages from fourteen to seventeen. The B.A.s of those days corresponded to the undergraduates of our time.

The arithmetic of the quadrivium was Greek (ratio, proportion and fractions), later with Arabic additions, including algebra. The geometry was Euclidean, and the astronomy Ptolemaic. The books used were in Latin, largely translations from the Arabian writings in which the Greek learning had been preserved and extended. In the period from the thirteenth to the sixteenth century notable Latin books on mathematics and science were written by the Oxford graduates Holywood (died 1246, author of long famous works on arithmetic and the sphere), Roger Bacon (1214-1294), Bradwardine (1290-1349) and Tonstall (1474-1559). Of course books were not easily available to students.

Roger Bacon called mathematics divine, the alphabet of all philosophy, and the proper basis of a liberal education, since it alone taught the student to distinguish truth from falsehood, and so fitted him to acquire other knowledge. The university statutes of Edward the Sixth, enacted in 1549, were based on like views, and introduced mathematics into the undergraduate course. But the Elizabethan statutes that succeeded them, reverting to the attitude generally maintained throughout the previous centuries, excluded mathematics, apart from arithmetic, from the undergraduate course. In any case, whether in the B.A. or the M.A. course, it seems likely that in the earlier centuries only a few studied the subject with any zeal; the majority gave their main efforts to logic, metaphysics and theology, then the chief avenues to place and fame. The neglect of mathematics did not endanger a candidate's mastership, at least at Cambridge, where no test of proficiency was imposed, though the M.A. candidate had to make a declaration that he had studied the subjects of the quadrivium.

The Renaissance.

Mathematics was in fact generally looked on as a mechanic art, a technical subject. With its sister subject astronomy, it was also associated with magic

* The address was illustrated by a collection of lantern slides (kindly assembled by Mr. F. J. Dyson) showing portraits of mathematicians and title-pages of old mathematical works.

† Sedleian Professor of Natural Philosophy, Oxford; Head of the Department of Mathematics at Imperial College, 1924-1946.

‡ In the early part of this address, I have drawn freely on Rouse Ball's *History of Mathematics at Cambridge*.

and astrology in the common view. Even serious and devoted mathematicians such as Kepler (1571-1630), yielding to popular demand, were fain at times to practise these arts for want of other support for their studies. Some scholars combined the profession of alchemy or medicine with that of mathematics.

Despite lack of official encouragement, however, men with a natural love and aptitude for mathematics did somehow cultivate it. During the sixteenth century it would seem that Cambridge became noted as a centre of mathematical studies; for example, the distinguished sixteenth century mathematical scholars, Tonstall and Recorde (1510-1558), migrated to Cambridge from Oxford; from 1501 onwards Cambridge had appointed university lecturers in mathematics, beginning with Roger Collingwood, author of *Arithmetica Experimentalis*. Roger Ascham was one of his successors.

At this time also some mathematical works began to be written in English—Recorde's arithmetic, entitled the *Grounde of Artes* (1540), his *Whetstone of Witte* (1556) on algebra, and other books on geometry, trigonometry, and astronomy; the first English translation of Euclid, by Billingsley, appeared in 1570.

The first Professorships of Mathematics.

The practical value of mathematics might be a cause of reproach among many at Oxford and Cambridge, but in Elizabethan times it began to be appreciated in the City of London. Mathematical lectureships were established there, and London became a centre of mathematical learning. For example, Edward Wright (1558 ?-1615), who constructed a theory of Mercator's projection, and aided navigation in many other ways, held a lectureship founded by the East India Company. In 1596, under the will of Sir Thomas Gresham (1519 ?-1579), founder of the Royal Exchange, Gresham's College, in the City, was instituted, with the first English professorships of geometry and astronomy. Henry Briggs (1556-1630), the inventor of common logarithms, was the first English professor of mathematics.

In 1619 Sir Henry Savile (1549-1622), Warden of Merton College, Oxford, who himself lectured on Euclidean geometry, followed Gresham's example and founded chairs of geometry and astronomy—the first *university* professorships in these subjects. Briggs resigned his Gresham chair to become first Savilian professor of geometry. Cambridge had no mathematical professorship till 1663, when the Lucasian chair was founded, its earliest holders being Barrow and Newton.

By about 1650 the mathematics read at the universities had been extended by the inclusion of logarithms, decimals, plane and spherical trigonometry, and Copernican astronomy. The brilliant, short-lived youth, Horrox (1619-1641), the best English mathematician and astronomer before Newton, has left a list of the 31 books he possessed, mainly astronomical rather than mathematical. They were chosen with great care, while he was a poor Cambridge student, mainly relying on his own reading. All were in Latin, and they ranged from ancient works by Herodotus, Pliny and Ptolemy to mediaeval and more recent books by Holywood, Mercator, Copernicus, Gassendi, Tycho Brahe, Kepler and Lansberg.

The Newtonian School.

In the second half of the seventeenth century, interest and activity in mathematics and natural philosophy reached a high pitch, both at Oxford and Cambridge, and also, with the foundation of the Royal Society, in London. The prestige of Newton and his discoveries, in calculus, mechanics, astronomy

and optics, was so great that his work was gradually introduced into the undergraduate course. By 1750, at Cambridge, mathematics and the Newtonian philosophy had come to dominate the studies at the university, and formed the basis of a new examination system, the Mathematical Tripos. This became the essential gateway to the Cambridge B.A. degree and any subsequent academic honours, and so continued for over a century : except that the noble and wealthy often obtained the award purely by favour.

By 1730 the textbooks in common use covered Euclid, Greek geometry of the conics, arithmetic, algebra, trigonometry, Cartesian geometry, calculus, mechanics, hydrostatics and astronomy. But the Cambridge school soon lost its vitality, and became for the most part only the transmitter of a tradition—a tradition at a far higher level, certainly, than ever before, but nevertheless out of touch with the vigorous mathematical life of the continent. The limitations of Newton's calculus notation, and his geometrical methods of exposition, were also retained, until about 1830 a new reforming school, among whom Herschel and Babbage were prominent, swept them away. Babbage helped to found (1812) the Analytical Society, formed, as he said, to advocate " the principles of pure *d*-ism as opposed to the *dot*-age of the university ".

Tradition and Reform.

In all subjects, and particularly in mathematics and science, in the universities as in the schools, both tradition and revision are alike necessary. The majority of teachers faithfully fulfil their task in transmitting the tradition they have received, with gradual improvements, of great value in the aggregate, which become embodied in textbooks. The minority, those who are exceptionally gifted, alert and wide-ranging in their studies, whether or not they extend their subject, perceive how to modify the content of the student's course and bring it up to a standard fit for the epoch. Time is gained by discarding matter of less importance for more important knowledge, or by improving proofs, so that comprehensive general theorems replace repeated proofs of particular cases ; in this way it becomes possible to introduce new branches of mathematics into the course without overloading the student. The task of revision is difficult and never-ending, one to be tackled consciously and experimentally, by teamwork among teachers, as this Association has so notably done, and is doing, for the school curriculum. The need is no less at the universities, and the opportunities are greater, because each university controls its own examinations. Life for the university teacher is apt to be over full and hurried, but time should be found, at intervals, for organised scrutiny and revision of the syllabus of the interlocking mathematical courses. Comparison of past university examination papers or syllabuses at intervals of ten or twenty years shows that this need does not go unrecognised.

University Examinations.

Our present system of university examinations has had a long growth from a very different beginning. For half a millennium the English examinations for the B.A. degree were entirely oral, consisting of disputations in Latin, in which a student, in the presence of his examiners, had either to assert and defend propositions, or to controvert them. For centuries these propositions were mainly religious, logical or metaphysical ; not till the eighteenth century did mathematics become prominent among them.

At Cambridge, in the first half of that century, an order of merit and a class began to be assigned on the basis of these disputations. Later this list came to be used only to divide the candidates into groups, leaving the order of merit in each group (with possible transfer from one group to another) to be

determined by an oral examination on more modern lines. The disputations were not finally discontinued till 1840. Towards the end, the Latin spoken in them became positively barbarous, as the language ceased to be habitually spoken at the university.

The Senate House examination at Cambridge, which began about 1725, and gradually overshadowed the disputations, was from its outset conducted in English. Originally the examiners proposed oral questions to classified groups of candidates. By about 1770 the order of the subjects of these questions was Euclid, trigonometry and algebra, passing on to mechanics, hydrostatics, apparent astronomy and optics, and then further, if the students proved capable, to conics, spherical trigonometry, higher algebra, and Newton's *Principia :* winding up with a few questions on ethics and philosophy. For the higher groups of students problems also were proposed, orally, and the students went aside to work out and write down their solutions. Then it began to be the custom to dictate the questions, one at a time, and to require written answers ; at first each new question was set when some student had solved the previous one, so that the more struggling candidates were continually interrupted. About 1790 the problem paper began to be printed, but not till 1827 was it prescribed that all the papers should be printed. Not until 1836 were all the candidates' scripts *marked* : previously the examiners " had partly relied on their impression of the answers given " ! The length of the examination was changed repeatedly, starting with one day and working up to five or six ; at one time it reached eight days. For many decades the examination was conducted in mid-winter in the unheated Senate House ; on one occasion the candidates found the ink frozen at their desks !

A position in the top class of the tripos, the wranglers, signified considerable mathematical ability and knowledge, but the minimum standard did not bear too hardly on those to whom mathematics is an uncongenial burden. As late as 1800 a position in the second class (the senior optimes) could be secured on a knowledge of Euclid's first two books, algebra to simple and quadratic equations, and the early chapters of Paley's *Evidences of Christianity*. At one time the lists included no less than eight classes, of which only the first three carried honours in the examination. The weaker candidates did not take any problem papers. Actually many of the problems were really book-work or easy riders. From 1828 onwards the candidates who did not seek honours (the so-called poll men) were given entirely separate papers, on a different syllabus, but their results were still announced, as late as 1858, as being in the fourth to the seventh classes of the mathematical tripos.

Competition with other Subjects.

The dominance of mathematics at Cambridge, as the sole avenue to a B.A. degree, continued until 1850 : the classical tripos was instituted in 1824, but at first it could be taken only by successful candidates in the mathematical tripos : from 1850 it offered another and separate path to a degree. Later other honours schools, of mental and moral science, natural science, and so on, were founded, and mathematics as a subject of study had to compete with others on roughly equal terms. At Oxford the honours school in mathematics combined with classics was instituted in 1800, and the two subjects were separated, and became independent means to a degree, as early as 1807.

It must certainly be considered a strange turn of fortune that placed mathematics in so dominant a position at Cambridge for so long a time : especially as the subject had for centuries been held in light esteem by most members of the university, perhaps at times being beholden for bare survival to the credit of having received much attention and respect from the ancient Greek philosophers. In the nature of things a marked power of appreciation

of mathematics, as of other arts, such as music, literature, or painting, is the possession of a fortunate minority, whether or not gifted with technical ability in the art. And though it may be argued that some knowledge of mathematics should be expected from almost all educated persons, it seems against Nature that it could for a century have been so forced upon *every* candidate for the university's warrant of learning, the B.A. degree.

Such dominance cannot wisely be regarded as a real privilege for whatever subject may possess it; at Cambridge it must certainly have condemned many unapt and unwilling pupils to dreary and useless hours, whose discomfort was shared also by their teachers, whether these likewise had been led by force of circumstance to profess an uncongenial subject, or were true lovers of it. Perhaps here I may be striking a sensitive chord in some of my hearers; and indeed my own experience has included moments that led me to reflect painfully on the unwisdom of imposing particular subjects, whether mathematical or literary, on all students without exception, either at school or university. Between some pupils and some subjects the repulsion seems so strong and insurmountable that means should be found, with whatever difficulty, for exceptional relaxation of general rules, at least where there is any compensation of excellence elsewhere.

It is at any rate good to be able to record that the Cambridge mathematicians yielded up their privilege and dominance without resistance, when at Cambridge the new stirrings of intellectual life roused the university from its eighteenth century slumbers and inspired a demand for freedom and equality for other subjects of study.

The Stream of University Trained Mathematicians.

The Cambridge tripos lists have been published since 1748, the Oxford mathematical honours lists from 1807. It is interesting to note from these lists the number of men who since those times have gained mathematical honours. At Cambridge throughout the second half of the eighteenth century the numbers fluctuated only slightly about an average of 40 per year; in the nineteenth century they increased rapidly to a maximum of 157 in 1840, and later declined under the free competition of other subjects; in the present century the numbers have slowly increased again, the average for the decade 1930-9 being 77. At Oxford the numbers have always been smaller, the annual average being about 20 over the whole period from 1830 to the present time.

During the nineteenth century other universities were founded in this country, and these, especially London, began to make appreciable contributions to the stream of university trained mathematicians. This stream also began to include women as well as men. The statistics of mathematical graduates in this country do not seem to have been collected in any convenient form, though it seems to me that a survey of such university mathematical teaching is a task well worth making. Mr. D. C. Gilles, who has helped me in obtaining the numbers above quoted, is undertaking such a survey at my suggestion, covering not only statistics of students but also of staffs, organisation, examinations, and subsequent careers. Some of the information in his tables is briefly summarised in Figs. 1, 2 (see folding plate). Fig. 1 shows from decade to decade the annual average number of honours graduates in mathematics produced by the principal universities of England, Wales and Scotland, those for the universities with an annual average of more than ten being shown separately. Fig. 2 gives the totals from all the universities, including those of Ireland. The numbers of men and women graduates are shown separately in each figure.

The numbers thus graphically given are substantially but not in all cases absolutely correct, as it has proved difficult to obtain information from one or two universities. The main features shown by the diagrams, apart from those for Oxford and Cambridge already mentioned, are the rapid rise of the London contribution, and the rough equality between men and women at this alone among all the universities.

Careers of Mathematicians.

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With the growth of elementary, secondary and university education in this country, the demand for university-trained mathematical teachers has expanded and continues to rise. Through the work of the mathematical teachers at secondary and public schools the boys and girls with special ability in mathematics are sifted with ever greater efficiency from the general community of children, and many are passed on to the universities. For them, their parents, and their school advisers it is important to know the nature of the careers nowadays open to mathematical graduates.

Teaching, mainly in schools but also in universities and institutions of intermediate rank (like technical colleges and institutes), must and will always claim a considerable share of the annual output of mathematical graduates. A few will adopt careers for which mathematics has no special relevance, except as part of their general education; for example, administrative, legal or business positions. But there is to-day an important and growing call for mathematical graduates for technical posts in which the work involves the constant *use* of the subject for practical purposes. This call comes from many sources: large manufacturing businesses, insurance companies, and Government departments need statisticians; industrial firms need mathematicians for design departments; research organisations, both in industry and in Government scientific departments, need mathematicians for special branches of their work, and also as members of teams composed of graduates in several different branches of science, *e.g.* physics, chemistry, and engineering; members of such teams incidentally influence one another so that they may tend to converge after some years to a common basis of knowledge, though with different aptitudes. Large government scientific organisations like the National Physical Laboratory and the Meteorological Office need mathematicians, together with graduates of other kinds, for work which may be either skilled practice, or research, or a combination of both. In biological work also the need for help from mathematicians is increasingly recognized.

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The time has come to enquire whether this variety of careers open to mathematicians should be reflected in some degree of corresponding specialization in our university mathematical courses. The autonomy of our universities in itself has naturally led to considerable variety in the courses they provide, due to the particular interests and educational ideas of their mathematical staffs. The question I propose relates to the obligatory part of the degree course in mathematics, and not to additional so-called advanced subjects on which optional examination papers are set. The obligatory course may lean either to the pure or the applied divisions of mathematics, or try to hold the balance evenly between the two. The field of mathematics is now so extensive that a broad and varied course can be selected with a strong bias towards either division. It seems to me desirable that intending students should have available for choice a number of courses, some "biassed", some general, so that different tastes, aptitudes and careers can be provided for. Where the mathematical staff is sufficiently large, such a variety of choice may be provided in a single university, but in any case the choice should be afforded by the whole group of our universities. Of course a good deal would be common to all such different courses.

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great value also in bringing about a direct personal association between the teachers and students, who can thus clear up with the teacher any difficulties found in the preceding or earlier lectures, and also get help in difficulties with the problems set: to a large extent these problems are chosen to be the simplest possible illustrations of the theory given in the lectures, so that the student can learn the theory by putting it at once into operation, before tackling harder examples that exercise and test his mathematical technique.

Further, prominence is given to general theorems from the earliest stages, saving much repetition and waste of time on particular cases. Vector methods are used from the outset, and the students are familiarized with three dimensions in all their work: likewise in their second year Cartesian tensors are introduced and much used.

Finally, developing an old tradition of the college, examinations are held twice yearly, at the end of each half session. Thus the whole examination is divided into four nearly equal parts, each of three or four papers, totalling fourteen in all. The intensity and wide range of the course is thereby mitigated, in the examination, because the students need to concentrate at each stage on only about four months' actual lectures.

In my own undergraduate period at Cambridge, those who took optional (Schedule B) subjects in Part II of the Tripos had twelve papers, occupying the mornings and afternoons of six consecutive days, and covering two or three years' work—a plan involving excessive strain, and quite needless for the purpose of ranging the candidates according to their ability and knowledge. The system at the Imperial College, which struck me as a doubtful novelty on first going there from Cambridge and Manchester, now, after long experience of it, appears admirable to me, and I should like to see it widely adopted. It achieves its purpose without imposing an undue burden on the students: and though the setting, printing and marking of as many as fourteen obligatory papers makes the conduct of the examination rather more expensive than if fewer papers were set, not only does it give fuller scope to each student to show his particular excellencies (as well as deficiencies), but it also reduces the overall average error of estimation and classification, from which no system of examination is free.

After completing the course and examinations in two years, usually a third year of approved study is required to fulfil the attendance requirements of the university. If this year is spent in the mathematics department, it is devoted either to advanced study and training in research, leading in many cases to the Ph.D. degree after yet another (fourth) year, or, in a minority of cases where the student is of less ability, to advanced study for the M.Sc. degree. Many students, however, after completing the mathematics course, transfer to some other department of the college, most often to physics, aeronautics or chemical technology, either to take a second honours degree (possible in two years in physics, but not in other subjects) or to do post-graduate work. Men with the double qualifications and experience thus gained have great value in industry and in government scientific work.

The proof of the pudding is in the eating, and it may be of interest to mathematical teachers in schools to learn something of the results of the course and examination system I have described. It must be remembered, however, that twenty years does not adequately test a system, because half the graduates produced will still be only a little way on in their careers. It must be recognized, also, that able students will make their way, though not with equal facility, whether their university course and training are good or not so good.

It is not easy to obtain complete statistics and particulars of the careers even of one's own students, but enquiries made with the aid of my colleagues

suffice to give the general picture. In the twenty years 1926-1945 the number of Imperial College candidates who passed in the mathematics B.Sc. examination has been 88, of whom 79 gained honours, 54 in the first class and 25 in the second class. Excluding the most recent graduates, some of whom are still undergoing further training in the college, and omitting those who have lost their lives, information has been collected regarding 58 old students. From this it appears that 33% have gone into the Government scientific services, 22% into industrial research and design establishments, 21% into university teaching, 16% into non-university teaching, and the remaining 8% have entered the church ministry, the army, business or other occupations. Many of those concerned are, of course, still so young that interchange between these classes may still occur, but the general picture will not, I think, be much altered.

In conclusion, if any of my hearers are interested in knowing more about non-teaching careers for their mathematical pupils, I should like to draw their attention to a valuable pamphlet *Industrial Mathematics*, by Thornton C. Fry, Mathematics Research Director to the Bell Telephone Laboratories, New York; in this pamphlet he discusses the nature of the work of mathematical specialists in industry, and their education, employment and supervision: next the industrial uses to which their mathematics is put: and thirdly the place of statistics in industry. S. C.

LONDON BRANCH.

OFFICERS for the year 1946 are: *President*: Dr. W. G. Bickley; *Chairman*: Dr. J. Topping; *Treasurer*: Miss A. K. Butler, Flat 2, 14 Park Hill Rise, Croydon; *Secretary*: A. J. G. May, 51 Wychwood Avenue, Luton.

The Annual General meeting on 19th January was followed by members' topics, when Mr. Bennett, of the Isleworth County School, read a paper on "The construction of the slide rule"; Miss Franklin, of Queen Elizabeth's Girls' School, Barnet, dealt with "Tabulation"; Mr. May, of Luton Grammar School, with "Converses"; and Mr. Moore, of Harrow School, with "Some technicalities in elementary mathematics".

At the meeting on 16th February Mr. Hooper, of the Bec School, opened a discussion on Sixth-Form mathematics. After referring to the war-time difficulties of London schools, and the consequent lowering of standards, the speaker dealt with the order in which he introduced various branches of the subject. Next he dealt with methods he had found most suitable. An interesting discussion followed, in which ways of dealing with groups at various stages were suggested.

16th March. "Statistics in the Sixth Form"—Professor H. Levy, of Imperial College. Considering an approach for Mathematics students by way of elementary probability, he advocated numerous simple experiments by which the student collected his own data. The speaker dealt briefly with sampling and random errors. In the discussion which followed Mr. Loveday outlined the Statistics course in use at Raynes Park County School.

A. J. G. MAY, *Hon. Secretary.*

1485. *Detective Logic.* They say that a murderer can't keep away from the scene of his crime. By that reckoning you'd have said that at least five people outside the household had had a hand in the bumping off of poor silly Clarice. —*Murder in Rockwater*, p. 93, by Margot Neville—but what's in a name!

SOME LETTERS FROM CHARLES HUTTON[1] TO ROBERT HARRISON[2].

By kind permission of the Reverend Angelo Raine, M.A., of York, the Owner of the Originals.*

Woolwich 13 *Janry* 1779.

My Dear Sir,

Nothing but the want of a proper occasion to trouble you could so long

* The thanks of the Association are due to the Reverend Angelo Raine for permission to print copies of these letters, and to Mr. Sidney Melmore for making the transcripts and sending them to the *Gazette*.

The letters, if of slight mathematical interest, give a vivid picture of an age of vigorous work and robust controversy. Some brief notes have been added concerning the more important persons mentioned in the letters. Mr. Melmore, Professor W. M. Roberts, Royal Military Academy, Woolwich, Professor E. H. Neville, and Mr. F. Puryer White, have supplied some of this material; other information comes from the *Dictionary of National Biography* (*D.N.B.*), the *Encyclopædia Britannica*, Hutton's *Mathematical and Philosophical Dictionary*, and R. C. Archibald, "Notes on some Minor English Mathematical Serials", *Mathematical Gazette*, April 1929.

The notes are intended to be simple identifications, and hence, in many cases, the titles of books and papers have been quoted without being checked by comparison with the originals.

[1] Charles Hutton (1737–1823) was born at Newcastle-on-Tyne, the youngest son of a pitman, who died when Hutton was five years old. The *Penny Cyclopædia* says that the family, a Westmoreland one, was connected by marriage with that of Sir Isaac Newton. Hutton himself as a boy worked in the pits, but turned schoolmaster at the age of eighteen, first in Jesmond, then a village just outside Newcastle; in 1760 he opened a mathematical school in Newcastle, where among his pupils were John Scott, later Lord Chancellor Eldon, and Bessie Surtees, with whom Scott eloped. In 1773 the Professorship of Mathematics at the Royal Military Academy was awarded to Hutton on the results of an open competition and examination: among the examiners were Maskelyne (3), Horsley (10) and Landen (17). In 1774 he became a Fellow of the Royal Society, and in 1779, Foreign Secretary of the Royal Society, as described in the first letter. From this post he retired within a few years, owing to disagreements with the President, Sir Joseph Banks. Banks was something of an autocrat, and Hutton's retirement has been described as virtual dismissal.

The *D.N.B.* refers to Hutton's "simplicity of habits and equability of temper", but his many controversies and some records of the R.M.A. suggest that he may have been a little cantankerous. At one time he was thought to take more interest in the houses which he built at Woolwich and sold to the Government, than in his duties at the "Shop".

Hutton wrote a number of books and papers, among which may be mentioned his large two-volumed *Mathematical and Philosophical Dictionary* (1796, 1795), and his *Mathematical Tables, containing common hyperbolic and logistic logarithms* (1785), notable for an historical introduction, learned and valuable if a little unfair to Napier. This book of tables is said to have inspired Maseres' (25) work on logarithms.

[2] Robert Harrison (1715–1802) was appointed Master of Trinity House School, Newcastle, in 1757; John Scott (Lord Eldon) and his brother William (Lord Stowell) were among his pupils. After his retirement he lived at Durham. According to a footnote to pp. 443–4 of a *History and Antiquities of Newcastle-upon-Tyne* (doubtless early nineteenth century, but title-page missing), Harrison was "a profound mathematician" with a valuable collection of mathematical books, and "understood Greek and Latin and had a competent knowledge of the Oriental languages". On his removal to Durham, "he was a member of a newspaper or coffee-room in that city, where he sometimes smoked a pipe. This indulgence being prohibited by a general order, he struck his name out of the subscription-list, adding, '*Vanished in smoke*'.... When in Newcastle, he was called *Beau Harrison*, and in Durham, *Philosopher Harrison*".

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Systematic information about the careers of men trained in mathematics at the universities is particularly difficult to obtain. As long as the mathematical tripos was the sole means to a degree at Cambridge, the stream of trained mathematicians included many who took it perforce, without any real personal interest in mathematics; able men of this type afterwards became lawyers, divines and public or private administrators. Many mathematical honours men became Fellows of colleges; some of these devoted their lives to mathematical teaching, either as university or college officials, or as private tutors: these mathematical "coaches" were of great importance to the work at both the old universities, and until the order of merit in the mathematical tripos was abolished, most of the best Cambridge mathematicians came under their influence. Other mathematical graduates took up teaching at public or private schools. A very few became professional mathematicians or scientists. Perhaps the earliest *public* scientific post in this country was that of Astronomer Royal, instituted in 1675, and since filled by no more than ten men, three trained at Oxford, and seven at Cambridge.

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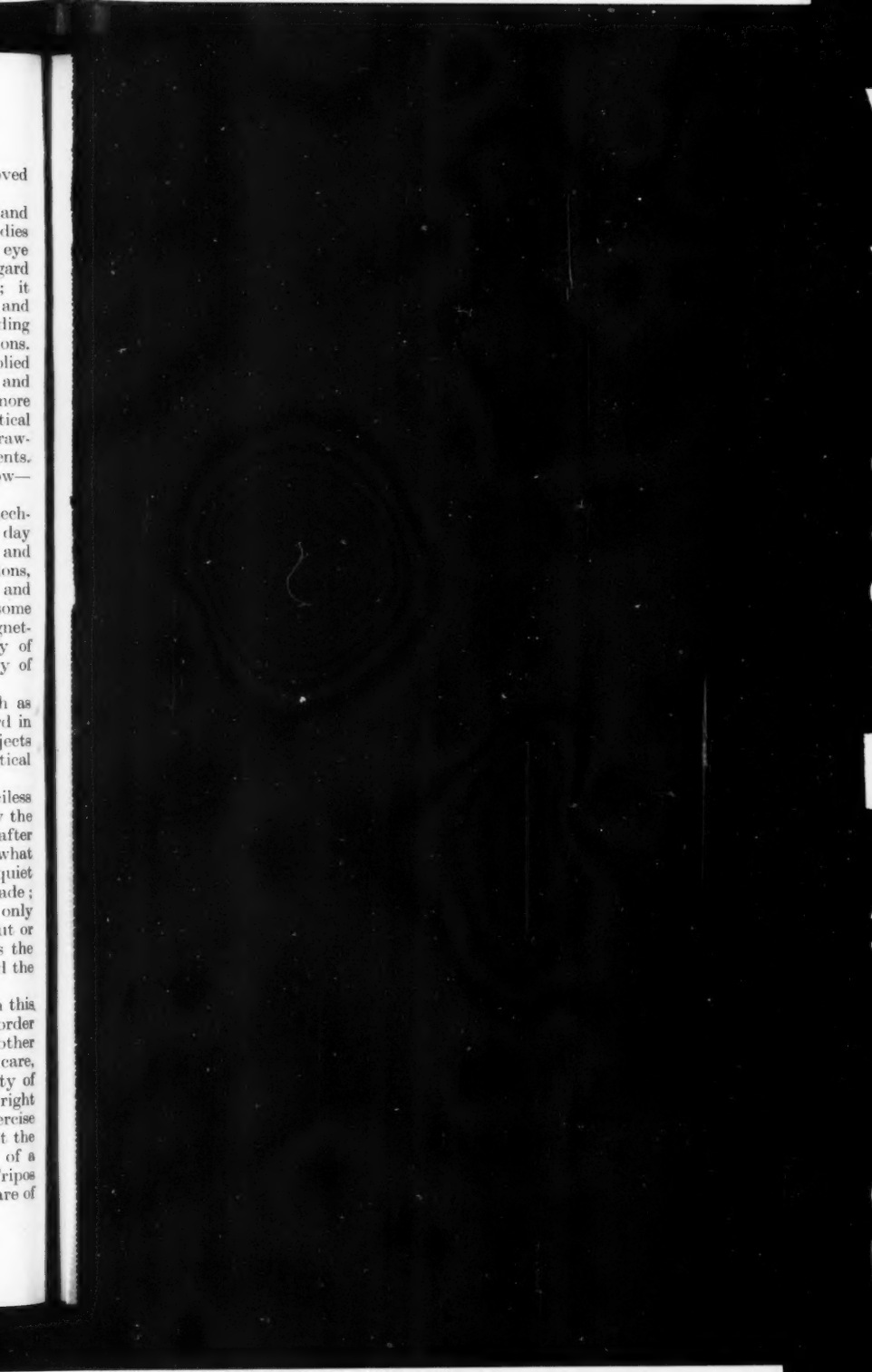

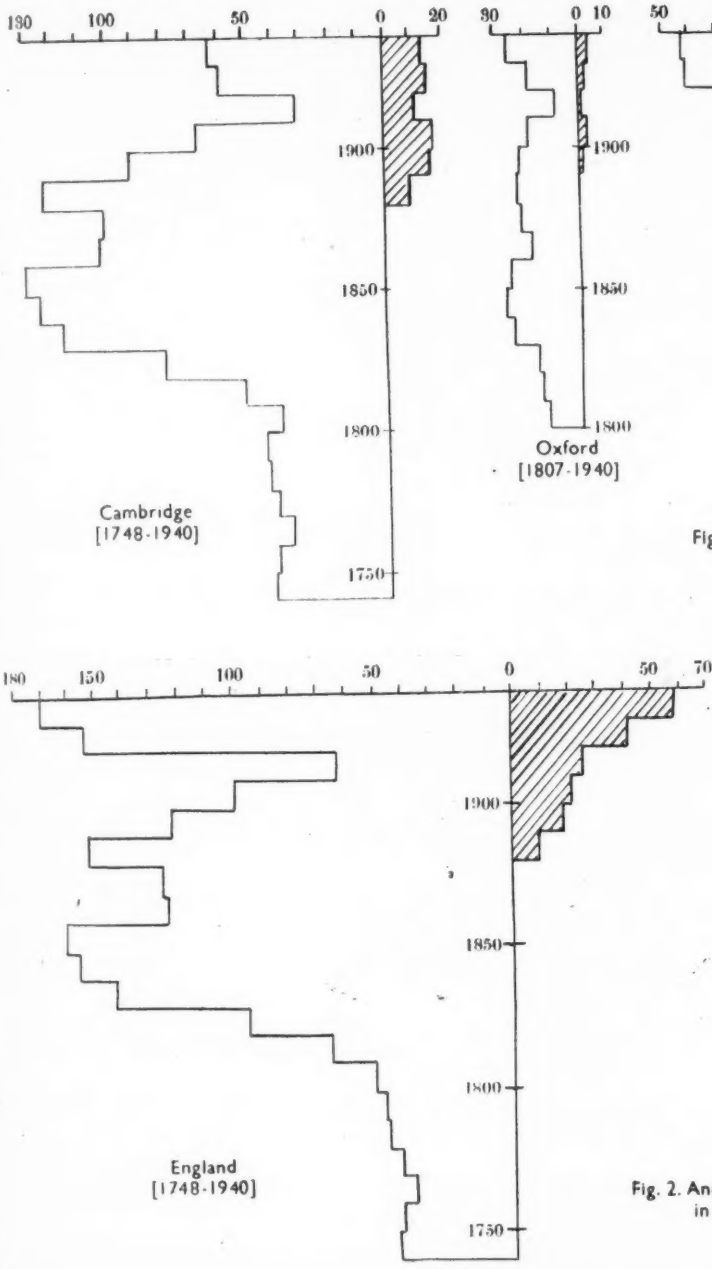
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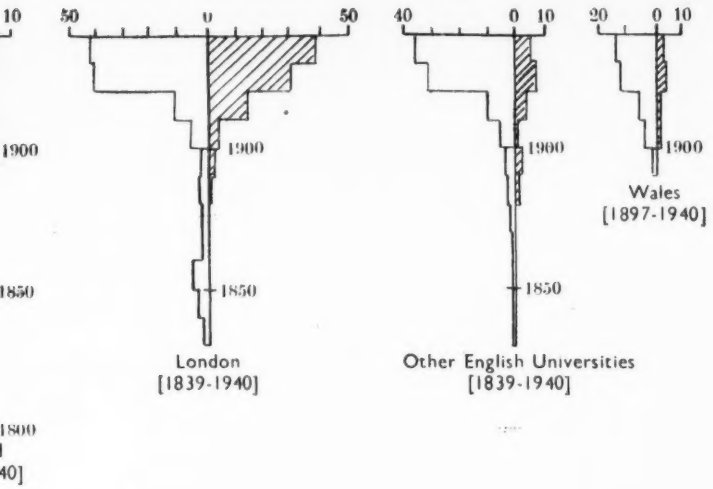
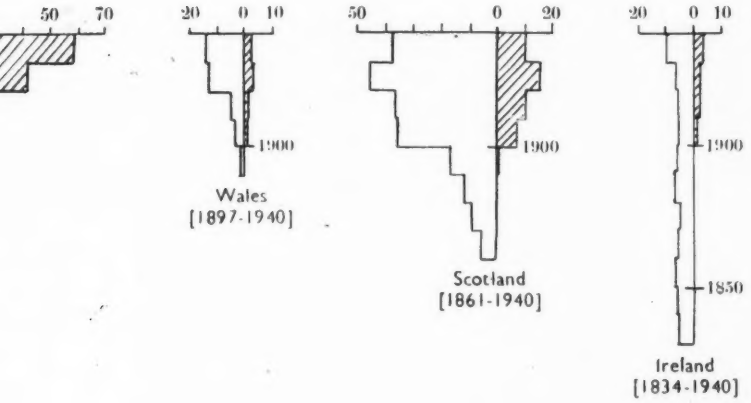


Fig. 1. Annual Average Numbers of Honours Graduates in Mathematics in each Decade for the Universities of England and Wales.

Men ☐ Women ▨



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Finally, developing an old tradition of the college, examinations are held twice yearly, at the end of each half session. Thus the whole examination is divided into four nearly equal parts, each of three or four papers, totalling fourteen in all. The intensity and wide range of the course is thereby mitigated, in the examination, because the students need to concentrate at each stage on only about four months' actual lectures.

In my own undergraduate period at Cambridge, those who took optional (Schedule B) subjects in Part II of the Tripos had twelve papers, occupying the mornings and afternoons of six consecutive days, and covering two or three years' work—a plan involving excessive strain, and quite needless for the purpose of ranging the candidates according to their ability and knowledge. The system at the Imperial College, which struck me as a doubtful novelty on first going there from Cambridge and Manchester, now, after long experience of it, appears admirable to me, and I should like to see it widely adopted. It achieves its purpose without imposing an undue burden on the students: and though the setting, printing and marking of as many as fourteen obligatory papers makes the conduct of the examination rather more expensive than if fewer papers were set, not only does it give fuller scope to each student to show his particular excellencies (as well as deficiencies), but it also reduces the overall average error of estimation and classification, from which no system of examination is free.

After completing the course and examinations in two years, usually a third year of approved study is required to fulfil the attendance requirements of the university. If this year is spent in the mathematics department, it is devoted either to advanced study and training in research, leading in many cases to the Ph.D. degree after yet another (fourth) year, or, in a minority of cases where the student is of less ability, to advanced study for the M.Sc. degree. Many students, however, after completing the mathematics course, transfer to some other department of the college, most often to physics, aeronautics or chemical technology, either to take a second honours degree (possible in two years in physics, but not in other subjects) or to do post-graduate work. Men with the double qualifications and experience thus gained have great value in industry and in government scientific work.

The proof of the pudding is in the eating, and it may be of interest to mathematical teachers in schools to learn something of the results of the course and examination system I have described. It must be remembered, however, that twenty years does not adequately test a system, because half the graduates produced will still be only a little way on in their careers. It must be recognized, also, that able students will make their way, though not with equal facility, whether their university course and training are good or not so good.

It is not easy to obtain complete statistics and particulars of the careers even of one's own students, but enquiries made with the aid of my colleagues

suffice to give the general picture. In the twenty years 1926-1945 the number of Imperial College candidates who passed in the mathematics B.Sc. examination has been 88, of whom 79 gained honours, 54 in the first class and 25 in the second class. Excluding the most recent graduates, some of whom are still undergoing further training in the college, and omitting those who have lost their lives, information has been collected regarding 58 old students. From this it appears that 33% have gone into the Government scientific services, 22% into industrial research and design establishments, 21% into university teaching, 16% into non-university teaching, and the remaining 8% have entered the church ministry, the army, business or other occupations. Many of those concerned are, of course, still so young that interchange between these classes may still occur, but the general picture will not, I think, be much altered.

In conclusion, if any of my hearers are interested in knowing more about non-teaching careers for their mathematical pupils, I should like to draw their attention to a valuable pamphlet *Industrial Mathematics*, by Thornton C. Fry, Mathematics Research Director to the Bell Telephone Laboratories, New York; in this pamphlet he discusses the nature of the work of mathematical specialists in industry, and their education, employment and supervision: next the industrial uses to which their mathematics is put: and thirdly the place of statistics in industry. S. C.

LONDON BRANCH.

OFFICERS for the year 1946 are: *President*: Dr. W. G. Bickley; *Chairman*: Dr. J. Topping; *Treasurer*: Miss A. K. Butler, Flat 2, 14 Park Hill Rise, Croydon; *Secretary*: A. J. G. May, 51 Wychwood Avenue, Luton.

The Annual General meeting on 19th January was followed by members' topics, when Mr. Bennett, of the Isleworth County School, read a paper on "The construction of the slide rule"; Miss Franklin, of Queen Elizabeth's Girls' School, Barnet, dealt with "Tabulation"; Mr. May, of Luton Grammar School, with "Converses"; and Mr. Moore, of Harrow School, with "Some technicalities in elementary mathematics".

At the meeting on 16th February Mr. Hooper, of the Bec School, opened a discussion on Sixth-Form mathematics. After referring to the war-time difficulties of London schools, and the consequent lowering of standards, the speaker dealt with the order in which he introduced various branches of the subject. Next he dealt with methods he had found most suitable. An interesting discussion followed, in which ways of dealing with groups at various stages were suggested.

16th March. "Statistics in the Sixth Form"—Professor H. Levy, of Imperial College. Considering an approach for Mathematics students by way of elementary probability, he advocated numerous simple experiments by which the student collected his own data. The speaker dealt briefly with sampling and random errors. In the discussion which followed Mr. Loveday outlined the Statistics course in use at Raynes Park County School.

A. J. G. MAY, *Hon. Secretary.*

1485. *Detective Logic.* They say that a murderer can't keep away from the scene of his crime. By that reckoning you'd have said that at least five people outside the household had had a hand in the bumping off of poor silly Clarice. —*Murder in Rockwater*, p. 93, by Margot Neville—but what's in a name!

SOME LETTERS FROM CHARLES HUTTON[1] TO ROBERT HARRISON[2].

By kind permission of the Reverend Angelo Raine, M.A., of York, the Owner of the Originals.*

Woolwich 13 *Janry* 1779.

My Dear Sir,

Nothing but the want of a proper occasion to trouble you could so long

* The thanks of the Association are due to the Reverend Angelo Raine for permission to print copies of these letters, and to Mr. Sidney Melmore for making the transcripts and sending them to the *Gazette*.

The letters, if of slight mathematical interest, give a vivid picture of an age of vigorous work and robust controversy. Some brief notes have been added concerning the more important persons mentioned in the letters. Mr. Melmore, Professor W. M. Roberts, Royal Military Academy, Woolwich, Professor E. H. Neville, and Mr. F. Puryer White, have supplied some of this material; other information comes from the *Dictionary of National Biography* (*D.N.B.*), the *Encyclopædia Britannica*, Hutton's *Mathematical and Philosophical Dictionary*, and R. C. Archibald, "Notes on some Minor English Mathematical Serials", *Mathematical Gazette*, April 1929.

The notes are intended to be simple identifications, and hence, in many cases, the titles of books and papers have been quoted without being checked by comparison with the originals.

[1] Charles Hutton (1737–1823) was born at Newcastle-on-Tyne, the youngest son of a pitman, who died when Hutton was five years old. The *Penny Cyclopædia* says that the family, a Westmoreland one, was connected by marriage with that of Sir Isaac Newton. Hutton himself as a boy worked in the pits, but turned schoolmaster at the age of eighteen, first in Jesmond, then a village just outside Newcastle; in 1760 he opened a mathematical school in Newcastle, where among his pupils were John Scott, later Lord Chancellor Eldon, and Bessie Surtees, with whom Scott eloped. In 1773 the Professorship of Mathematics at the Royal Military Academy was awarded to Hutton on the results of an open competition and examination: among the examiners were Maskelyne (3), Horsley (10) and Landen (17). In 1774 he became a Fellow of the Royal Society, and in 1779, Foreign Secretary of the Royal Society, as described in the first letter. From this post he retired within a few years, owing to disagreements with the President, Sir Joseph Banks. Banks was something of an autocrat, and Hutton's retirement has been described as virtual dismissal.

The *D.N.B.* refers to Hutton's "simplicity of habits and equability of temper", but his many controversies and some records of the R.M.A. suggest that he may have been a little cantankerous. At one time he was thought to take more interest in the houses which he built at Woolwich and sold to the Government, than in his duties at the "Shop".

Hutton wrote a number of books and papers, among which may be mentioned his large two-volumed *Mathematical and Philosophical Dictionary* (1796, 1795), and his *Mathematical Tables, containing common hyperbolic and logistic logarithms* (1785), notable for an historical introduction, learned and valuable if a little unfair to Napier. This book of tables is said to have inspired Maseres' (25) work on logarithms.

[2] Robert Harrison (1715–1802) was appointed Master of Trinity House School, Newcastle, in 1757; John Scott (Lord Eldon) and his brother William (Lord Stowell) were among his pupils. After his retirement he lived at Durham. According to a footnote to pp. 443–4 of a *History and Antiquities of Newcastle-upon-Tyne* (doubtless early nineteenth century, but title-page missing), Harrison was "a profound mathematician" with a valuable collection of mathematical books, and "understood Greek and Latin and had a competent knowledge of the Oriental languages". On his removal to Durham, "he was a member of a newspaper or coffee-room in that city, where he sometimes smoked a pipe. This indulgence being prohibited by a general order, he struck his name out of the subscription-list, adding, '*Vanished in smoke*'.... When in Newcastle, he was called *Beau Harrison*, and in Durham, *Philosopher Harrison*".

have prevented me from having the pleasure of addressing you. I hope you have enjoyed good health & every other happiness during the interval. As to my own health, it is pretty well; but I am here almost as recluse as a hermit, being almost single in my studies & manner of thinking in this place, my nearest neighbour in these respects being the Astron. Royal (3) at Greenwich with whom I have the honour to be on very good terms. I herewith send you a small assortment of *American* seeds some of them mixed with a little of that sacred soil so nourishing to the seeds of freedom! there is little more of it that we can now call ours! These seeds are the joint collection (chiefly in the neighbourhood of New York) of our friend Williams (4) & a Cap. Blomefield (5) who has just brought them over with him to be sent to the Durham hermit. This latter gentleman (B) is a very intimate acquaintance of W's, & intends to do himself the pleasure to write to you soon, being furnished by W. with your address for that purpose. W. is now a full Captn. he having lately acceded to a company, & would have come over at this time to Europe to join it but has been tempted to remain in A. by the temporary office of Major of Brigade which may be worth near 20 s. pr Day to him. He was become very fat & had his health well till lately that a dangerous fever seized him, from which he is just happily recovered. He sometimes I find snatches an opportunity, in spite of stern Bellona & the furies, of sacrificing to Minerva & the Muses, & by turns amusing himself & others with Mathematics, Music, Poetry, Painting, &c & diffusing rational entertainment all around him. If any plays or entertainments are to be represented, W. directs the machinery, paints the scenes, points out the subjects, teaches the Dramatis personae & the band of Music, writes and speaks the prologue & epilogue, acts the principal part &c, &c. He has lately been contriving or improving a circular sliding rule, has divided & made one himself, taught an Instrument Maker in N.Y. to execute them for his Brother Brigs. & has printed there the description & use of it. In short he is all in all.

I herewith also send you a few sets of three papers I have had printed in the Philos. Transactions, one of which brought me the honour of Sir Godfrey Copley's annual prize Medal (6), I have also sent as many Copies of the Presi-

[3] Nevil Maskelyne, Astronomer-Royal 1765–1811; famous as the founder of the *Nautical Almanac* and for his Schiehallion experiment (1774). Hutton was selected to determine, from Maskelyne's observations, the estimated mean density of the earth; several references to this work appear in these letters. Hutton at this time had as a colleague Bonnycastle, chiefly remembered for *The Scholar's Guide to Arithmetic*, and later was joined by John Evans, F.R.S., so perhaps his loneliness was thereby relieved. See also under (1).

[4] "Our friend Williams." This is probably Edward Williams, R.A. (see Kane's *List of R.A. Officers*). He became a Captain in 1775, and was on the staff of the Governor of New York, 1778–1780, and died, a Lieutenant-Colonel, in 1793. The reference will be understood by remembering that this country was then in a perilous position. We were at war with the revolted American colonies, and with half Europe; fifteen months earlier Burgoyne had surrendered at Saratoga, two years later Cornwallis capitulated at Yorktown. In the summer of this year 1779, the combined fleets of France and Spain swept the Channel and threatened Plymouth.

[5] Blomefield was the very celebrated Gunner of his day, who went to America in 1776 as a Brigade Major, and was wounded before Saratoga. In 1780 he was made Inspector of Artillery and of the Brass Foundry (in the Arsenal). Here he carried out a drastic purge, condemning 496 pieces in one year. From Blomefield dates the high character of British cast-iron and brass pieces (see Kane, *loc. cit.*). At Woolwich the degree of fame of the big men in the R.A. (and Woolwich soldiers generally) is marked by the names of public-houses and roads; Bloomfield Road is a main road in Plumstead.

[6] The papers (1776–1778) for which Hutton was awarded the Copley Medal of the Royal Society were on "The force of exploded gunpowder and the velocities of balls"

dent's speech made at the presenting of the Medal. After keeping a set yourself, let me request the favour of you to send a set to Mr. Emerson (7), to Mr. Geo. Anderson N.Castle, to Dr. Rotheram (8), Mr. Lowthian (9), & such other persons as you may think proper together with my Compts to each gentleman. I wished to have had another paper of mine ready to have sent much larger than all the others, viz the Computations for determining the general attraction of matter, by which it appears that the mean density of the earth is to that of common stone, in the ratio of 9 to 5 nearly ; but as those sheets are not yet printed, they must be sent at another opportunity.

On the resignation of Dr. Horsley (10), one of the two reading & minuting Secretaries to the R.S. two candidates were put in nomination for his office, viz. the Rev. Mr. Maty (11) (son of the late Dr. Maty many years secretary) & your hble servt. It soon appeared so clear both to my friends & myself that the majority of the members usually attending the meetings seemed to give the preference to us in their wishes, that we contented ourselves with doing very little more then announcing my intention. My Competitor's party, on the contrary, went about, with every possible means of interest, to every member, & by one means or other surprized a great number into promises to vote for him, by which means it happened that we were left in a considerable minority by the votes of many against me whose wishes were for me. And with the declaration of all the Society that the office should be quite at my service the next opportunity if I continued in the mind to accept it. They also did me the honour to elect me into the Council for the ensuing year, & have presented me with what is sometimes called the Latin Secretaryship & sometimes the Foreign Secretaryship. The duty of it consists in trans-

[7] William Emerson (1701–1782) was born at Hurworth, near Darlington ; after a short spell of teaching, he devoted his time to mathematics, living at Hurworth on a small income left to him by his father. Emerson wrote books on most branches of mid-eighteenth-century mathematics and natural philosophy, the earliest being his *Doctrine of Fluxions*. He had a big reputation in his day, too big, according to De Morgan ; but he declined to become F.R.S. The account of Emerson in Hutton's *Dictionary* exhibits him as a hot-tempered and odd-mannered man ; possibly he and Hutton were a pair. Hutton says : " His manner and appearance were that of a rude, and rather boorish countryman ; he was of very plain conversation, and indeed seemingly rude, commonly mixing oaths in his sentences, though without any ill intention ". Emerson was a determined walker, hating to ride in a carriage, and, again according to Hutton, " He often walked up to London when he had any books to be published ". As regards his books, he had a habit of omitting his name from the title-page ; if his prefaces were not signed, half his books would be anonymous.

[8] Rotherham or Rotheram was a common surname in the South Tyne area ; and the famous dissenting academy at Kendal was run by Caleb Rotheram. But this Rotheram is probably John, Professor of Natural Philosophy at St. Andrews, born at Hexham, educated at Newcastle Grammar School at the time when Hutton was teaching in Newcastle. John Brand, *History and Antiquities of the Town and County of Newcastle-upon-Tyne*, I, p. 447, footnote, quotes from the *Newcastle Courant* of September 22, 1770, a notice of the publication of " Dr. Rotheram's *Philosophical Enquiry* &c concerning the waters in or near Newcastle ". The *D.N.B.* gives among his works *A Philosophical Inquiry into the Nature and Properties of Water*.

[9] Lowthian is probably Samuel Lowthion ; according to Brand (*loc. cit.*) " a popular preacher, and published several sermons. He died of the gout in his head, Nov. 17th, 1780 ".

[10] Samuel Horsley, well known for his theological controversy with Priestley and as the editor of the only edition of Newton's Works. See also under (1).

[11] P. H. Maty took a strong part in the dispute between Banks and Hutton. Rather surprisingly in view of this letter, Maty seems to have been on Hutton's side, though this may have been rather as against Banks than for Hutton.

lating into English such papers as are sent to the Society from abroad by Foreigners, those papers being usually written in French or Latin. They are translated in order to their being read in English to the mixed Company at the ordinary meetings of the Society. I have finished one very large translation already, & as I have lately been not inattentive to such studies, I hope to acquit myself to satisfaction in the business.

I mentioned Mr. Emerson, pray how does he do? I hope as well as can be expected for his age; present him with my best wishes for his happiness, & if you please mention to him my doubts concerning the truth of what he asserts in the Schol. near the bottom of pa. 433 of his Miscellanies, about Bernouilli's two probs, that he may consider what is proper to be done against a new edition if it be printed. I can prove the truth of the conclusions of the two solutions in I think a manner that will satisfy him, & will communicate it to him if he think it worth his while, or if he be not himself already convinced of the truth of them. This conduct indeed concerning these two probs surprizes me much: for if he thought the solns wrong, he ought to have given true ones if they admit of them; but if not, the probs & false solns ought not to have come into his book: & his assertion that "the solutions are wrong, for both the quantities are infinite" is very extraordinary; for he well knows that the sum of a series is not always of an inf. magnitude when the number of terms is inf. & if he thought the sum inf. for any other reason, he should have shewn it. Possibly he does not yet know that we are not now to expect any more Fame's Palladins (perhaps you will say that is no loss), that terrible & ugly fellow who with such long strides pursued poor Tristram into France, having lately stretched the great Capt. Heath in terra.

I am, Dear Sir, Yours most sincerely & affectionately,

CHA. HUTTON.

WOOLWICH, 4 *Aug.* 1779.

DEAR SIR,

Two or 3 days since I sent to the Durham waggon a parcel for you. It contains 7 copies of a paper I have lately had printed in the Philos. Trans. which I desire the favour of you to distribute to the same Gentlemen to whom you delivered the copies of my former papers. This last paper is a computation to ascertain the mean density of the whole globe of the earth; a very important problem, and a work of immense labour. You will perceive what the Data & Conclusions are, which I believe are not far from the truth, tho' it be the 1st experiment of the kind. I have accompanied them with one copy of the paper containing the astronomical observations, which is intended for your own use, & is a proper companion to my paper. I have no letter from our friend Williams since I last wrote to you, but I hear he is well at New York. Mrs. Poole lives at Dover, & I believe is very well. Mr. Thos. Williams (their brother) is lately come from India, in a very bad state of health; he also at present I believe is at Dover. Harry's (my son) (12) station is at Coxheath Camp. My own health &c pretty well as usual, so have nothing to tell you farther concerning myself, unless I mention that the University of Edinburgh have just now done me the honour to grant me a Diploma for the Degree of Dr. of Laws!

I forgot to mention above, that I could wish you to give one of the Copies of the paper to Mr. Hartley of Yarm, & to present my respects to him.

[12] "My son Harry" rose to the rank of Lieutenant-General, and died in Ireland, 1827. He must have been well thought of, for when in 1814 a large number of Gunners were retired, he received a pension of £700 per annum. Coxheath Camp was one of the Militia camps established during the American war, at a time when a French invasion of this country seemed likely.

Mr. Muller (13) was himself in an error, instead of Mr. Robins (14), concerning the subject of his censure of him. And now for my friend Mr. Emerson. In the first place I must say that his whole conduct concerning the 2 probs of Bernouilli quite amazes me, nor could I easily have believed that a man of his knowledge & good sense could have committed so many inconsistencies & improprieties. I agree with you that the problems are little other than Algebraic Juggles, but still their solutions are not false, nor can the two quantities in the probs have any other possible meanings than one each, if letters & symbols have any determinate signification at all notwithstanding what Mr. E. says about the duplicity of the meaning. I like Bernoulli as little perhaps as Mr. E but in endeavouring to expose the former I was sorry that the latter exposed himself, for the method he made use of to do it appears to me highly improper even tho' B. had been in an error concerning these solutions. I repeat it therefore, it was with reason that I thought his conduct *extraordinary* & that it *surprised* me, for I hold it utterly unworthy the character he has long borne for mathematical knowledge. It has also given me some concern to find Mr. E. receive with evident ill-humour the well-intended hints of his friend. My time is too much employed for me to concern myself with any person whom I regard not, & I wished to give him an opportunity of correcting his oversights in a public manner himself & prevent his enemies from doing it for him in a disagreeable manner. And it is not impossible some such thing may happen, for since I wrote to you, I have received from a Diary Correspondent a request to propose these very probs in that work on purpose to correct Mr. E's oversight ; so that possibly such a thing may be sent to some other place where it may [be] used against him. But as I do not desire to trouble his repose, I wish not to communicate to him anything more on this subject, what I have farther to say on it being for your own satisfaction & at your own request. I shall just remark however that I had another motive for mentioning these probs to him, which was that as he possibly & unfortunately might not have a great number of years yet to live, & that as I foresaw I might have some care of any future editions of his books, I was desirous of procuring from himself any corrections or hints of improvements which he might wish to have made ; which was a motive that had his honour for its object. And indeed I have already by his own consent the care of a new edition of his Algebra which is in the press.

Let us first take an example in numbers to each of the probs viz., in the 1st prob. take $a = 6$, & let us approach to the sum of the series by beginning at the farther end of it & tracing it backwards. Now the last term of the series is $\sqrt{a}$ or $\sqrt{6}$ which is less than 3, i.e. $\sqrt{a} < 3$; take in the last term but one, then $a + \sqrt{a} < 6 + 3 < 9$, or $\sqrt{a + \sqrt{a}} < 3$; take in the antepenult. then $a + \sqrt{a + \sqrt{a}} < 6 + 3 \leq 9$, & $\sqrt{a + \sqrt{a + \sqrt{a}}} < 3$ and thus by continually adding another term the sum will be always less than 9, & ∴ the root of the sum less than 3 ; so that the value of the series approaches continually to the No 3, which is its limit or the sum of the inf. series. So that in this instance the

[13] Probably John Muller, Professor of Fortification at the " Shop " and the first of the civilian staff at its foundation in 1741. " The father of engineering for 40 years in Britain."

[14] No doubt Benjamin Robins, a versatile performer in many fields, mathematics and gunnery in particular. He was an unsuccessful competitor for the Professorship obtained by Muller at the R.M.A. Robins invented the ballistic pendulum (about 1740), and was the first man to determine the velocity of a shell. Mathematically, he may be remembered for his *Discourse concerning the Nature and Certainty of Sir Isaac Newton's Methods of Fluxions, and of Prime and Ultimate Ratios*, and other papers on the same topic.

series is not only finite but = to B's theorem,

for $x = \frac{1}{2} + \sqrt{a + \frac{1}{4}}$ is $= \frac{1}{2} + \sqrt{6\frac{1}{4}} = \frac{1}{2} + \sqrt{\frac{25}{4}} = \frac{1}{2} + \frac{5}{2} = 3$.

Again take $a = 1$, & $b = 8$ in the 2nd prob. & trace the series back as before. The[n] $\sqrt{b} = \sqrt{8}$ is < than 3, & $a \times \sqrt{b} = 1 \times \sqrt{8}$ less than 3, conseq. $\sqrt{a + \sqrt{b}} < 2$, i.e. the last couplet is less than 2; repeat this operation, then $b \times \sqrt{a} \times \sqrt{b} < 16$, & $\sqrt{b \times \sqrt{a} \times \sqrt{b}} < 4$ & $\sqrt{a + \sqrt{b \times \sqrt{a + \sqrt{b}}}} < 2$ i.e. the two last couplets less than 2; & the same thing happens after ever so many couplets; ∵ the series approximates infinitely to the no 2, which is the value of B's theorem

$$x = \sqrt[3]{aab} = \sqrt[3]{1 \times 8} = 2.$$

Proceed we now to the investigations of the general theorems. And first for the 1st prob. Let $s^2 - s = a$, then shall s be the sum of the inf. series $\sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a}}}}$ &c. For since $s^2 - s = a$, or $s^2 = a + s$, or $s = \sqrt{a + s}$, ∵ $s >$ (greater than) $\sqrt{a}$; add a to each, so shall $s + a > a + \sqrt{a}$, but $s^2 = s + a$, ∵ $s^2 > a + \sqrt{a}$, or $s > \sqrt{a + \sqrt{a}}$; add a again, then

$$s + a \text{ or } s^2 > a + \sqrt{a + \sqrt{a}} \quad \& \quad s > \sqrt{a + \sqrt{a + \sqrt{a}}};$$

then adding & extracting again, it will be $s > \sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a}}}}$. And thus the series approaches continually to the value of s which is its limit. Since then $s^2 - s = a$, ∵ $s = \frac{1}{2} + \sqrt{a + \frac{1}{4}}$ = the sum of the inf. series.

Again in the 2nd prob. let us begin the operation at the farther end of the series, & trace its value backwards towards the beginning by actual multiplications & extractions. The last term of the series being $\sqrt{b}$ or $b^{\frac{1}{2}}$, let it be multiplied by a & the root taken, so shall $a^{\frac{1}{2}} b^{\frac{1}{4}} = \sqrt{a + \sqrt{b}}$; mult. by b & take the root, then by a & take the root, so shall $a^{\frac{5}{8}} b^{\frac{5}{16}} = \sqrt{a\sqrt{b\sqrt{a}\sqrt{b}}}$; repeat the operation so shall $a^{\frac{21}{34}} b^{\frac{21}{64}} = \sqrt{a\sqrt{b\sqrt{a\sqrt{b\sqrt{a}\sqrt{b}}}}}$; in like manner the next operation will give $a^{\frac{85}{128}} b^{\frac{85}{256}}$; the next $a^{\frac{341}{512}} b^{\frac{341}{1024}}$; &c. where it is evident that the exponent of a is always double of that of b; that the exponent of a after any one extraction is always found by adding $\frac{1}{2}$ to $\frac{1}{4}$th of its exponent in the preceding expression; & that the exponent of b is found by continually adding $\frac{1}{4}$ to $\frac{1}{4}$th of its former one. But in all the expressions the exponent of b is a vulgar fraction whose denominator is always 1 more than three times its numerator, so that when its numerator is n its denominator is $3n + 1$, & this property will continue without end, for $\frac{n}{3n+1}$ being any exponent of b, by the construction of the series the next exponent will be $\frac{1}{4} + \frac{1}{4}$ of

$$\frac{n}{3n+1} = \frac{1}{4} + \frac{n}{12n+4} = \frac{4n+1}{12n+4}$$

where the denom. is still = 1 + 3 times the numerator. Now when the series is supposed to be infinitely continued, n is infinite, & then the exponent $\frac{n}{3n+1}$ is $= \frac{n}{3n} = \frac{1}{3}$ the expon. of b; & ∵ $\frac{2}{3}$ the expon. of a; conseq. the value of the inf. series is $a^{\frac{2}{3}} b^{\frac{1}{3}} = \sqrt[3]{aab}$, as in the solution. Hence it appears that the sum of the series are finite & rightly assigned by Bernoulli.

I am, my Dr Sr with much respect & gratitude

Your much obliged & obedt Servt

CHA. HUTTON.

WOOLWICH, 31 *May* 1780.

DEAR SIR,

I sit down to ask how do you do, without having anything material to tell you on my part, things here being in statu quo, & I have not had a letter from our friend Williams this 12 months. I sometimes however hear of him, & know that he is well, tho' he has had a bad & long fever since my last receipt.

How does the old Gentleman at Hurworth? Some months since I sent to him 7 Copies of an additional paper of mine on *the place of greatest attraction* on the side of a hill. I sent them by a Mr. Wright, a near relation of his, who was formerly a Coal Viewer about Tanfield &c. but has been for some years past in Scotland, where he was returning from London after being up on some business. The parcel was accompanied with a letter to Mr. E. in which, after keeping one of the pamphlets himself, I desired him to send the rest to you to dispose of as the former ones had been. Have you received them, or heard of them? I often meet with Newcastle people in London, but the sight of my much esteemed friend H. would afford a real pleasure indeed. I met young Mr. Smith lately: he is just going out to India. I am sorry to learn by him that the Grecian Lady is in danger of losing that language again, or at least does not pursue it, her mother having assumed the whole direction of her, for fear her learning should spoil her Marriage! We have just come out the 2nd Vol. of Horsley's Newton (the Cuts all done by Beilby of N.C.). Lately publish'd also a most excellent work on Infinite Series, by a Mr. Henry Clarke (15) (not my friend Sam. (16)). Mr. Landen has (17) just finished from the press a 1st Vol. of Mathematical Memoirs. He intends

[15] Henry Clarke, Professor of History, Geography and Experimental Philosophy at the Military College, Great Marlow (afterwards Sandhurst), author of several mathematical books. This book, *A Dissertation on the summation of infinite converging series with algebraic divisors, exhibiting a method not only intirely new, but much more general than any other which has hitherto appeared on the subject* (1779), is "translated from the Latin of A. M. Lorgna, Professor of Mathematics in the Military College of Verona" and has "illustrative notes and observations" and "an appendix, containing all the most elegant and useful Formulae which have been investigated for the Summing of the different Orders of Series". It is dedicated, in the florid style of the time, to Hutton. Clarke declares in his preface that Lorgna's methods are to be preferred to Simpson's, "both with respect to the perspicuity of his investigations, and the generality of his conclusions". Landen (17) considered that due credit had not been given to Simpson's work, and published some tracts on the subject, about which Hutton remarks: "Mr. Landen did not shew less mathematical skill in explaining and illustrating these theorems, than he has done in his writings on original subjects; and that the authors of them [De Moivre, Stirling, Thomas Simpson] were as little aware of the extent of their own theorems, as the rest of the world were before Mr. Landen's ingenuity made it obvious to all".

[16] "My friend Sam" Clark started the *Diarian Repository* as a rival to Hutton's *Diarian Miscellany*; this, and other actions of Clark's, provoked Hutton considerably. The *Repository* was described as edited by a "Society of Mathematicians", but this appears to have been a Society with one member only, "my friend Sam".

[17] John Landen, an estate agent, wrote on many mathematical topics, and is best remembered for his work on elliptic arcs, the genesis of the familiar Landen transformation of elliptic functions. As regards the *Mathematical Memoirs*, the first volume contained results on the motion of a body under no forces; attacking the general problem later, Landen obtained results differing from those of d'Alembert and Euler, which "made him long dubious of the truth of his own solution" (Hutton). Eventually he published it, and was thereupon engaged in a controversy in which Wildbore (19) took part. In spite of a serious and painful illness, Landen was able to prepare a second volume of the *Memoirs*, and a copy of the published work was placed in his hands the day before his death, which occurred in 1790. See also under (1).

going on if health permit, which is in but a delicate & doubtful way. His death would be indeed a great public loss, as he is a most excellent Mathematician & very worthy Man.

Old Peat (18), the author of the Gent. Diary, is dead ; & I have procured that work for the rev. Mr. Wildbore (19), a near neighbour of a revd Mr. Geo. Walker (20) now at Nottingham, whom you may have formerly seen at Mr. Lowthian's or about N.C. after which he lived some time at Durham (being the Dissenting Minister) whence he sent some excellent probs & solutions to the Ladies' Diary under the Signature of *P.M. of Durham*. I sometime hear the highest character of a Dr. Scott (21) of Oxford, who I think was an old pupil of yours in N.C. He is at present a Candidate (on the Whig Interest) for Member for Oxford University, as is also (the persian) Mr. Jones (22), author of the Synopsis Palmar. Mathes. & editor of Sir I. N's Miscellaneous pieces. There is a 3rd Candidate, but I know not who. It is said Dr. Scott is safe for one.

Hoping you enjoy a good State of health, I remain

Your most sincere & affectionat hble Sert

CHARLES HUTTON.

WOOLWICH, 19 *March* 1781

DEAR SIR,

I take the opportunity of my son's going thro' Durham to send the papers accompanying this. He is going down to Scotland where he is ordered on duty, & has directions to wait upon you if the Stage Coach stop long enough in Durham to permit him so to do. I don't find that I have much new intelligence to give you. Our friend W. was very well at New York pr. last Accots & in a way of making money, so the longer the conquest of America is protracted from year to year, the better for him & most others concerned in it. How is Mr. Emerson? I know not of any thing in the Transactions about the rotation &c of Venus, but there was lately read a paper at the Society, from a Mr. Herschel (23) at Bath, concerning his observations of the

[18] " Old Peat ", Thomas Peat, " Writing-Master and Teacher of the Mathematicks at Nottingham ", was a founder of the *Gentleman's Diary or the Mathematical Repository* in 1741 ; he was connected with the *Diary* till his death, editor 1757–1780. For Hutton's close connection with the *Diary*, see Archibald, *Gazette*, XIV, pp. 379–400.

[19] The Rev. Charles Wildbore, of Sulney, Notts., edited the *Diary* till 1802. See also under (17).

[20] The Rev. G. Walker was born at Newcastle and educated at the dissenting academy at Kendal under Caleb Rotheram. Among his works are *A Treatise on the Conic Sections. In five books.* (1794), though in spite of the title-page, this is only Book I, nothing further being published : and *On the doctrine of the sphere, in six books : to which is added an appendix, containing the solution of a problem for ascertaining the latitude and longitude of a place, together with the apparent time* (1777).

[21] William Scott, later Lord Stowell, a former pupil of Harrison's (2), D.C.L. Oxford 1779, is said by the *Encycl. Brit.* to have contested the Oxford University seat unsuccessfully in 1780. If his brother John, later Lord Eldon, was taught by both Harrison and Hutton (1, 2) this reference seems curiously vague, but the identification is probably correct.

[22] Hutton seems here to confuse father and son. William Jones, author of *Synopsis Palmariorum Matheseos* and editor of the Newton tracts, died in 1749. His younger son, Sir William Jones, was the linguist and Persian scholar, and to him this sentence must refer ; he was a candidate but withdrew before the election.

[23] " a Mr. Herschel at Bath " is undoubtedly William (later Sir William) Herschel, " the most illustrious astronomer of the latter half of the 18th century " (Rouse Ball).

rotation &c of the planets, which paper I believe will be printed in the Transacs. The Society have begun their meetings at their new apartments at Somerset House, which are more spacious & elegant than the former. The account of the last voyage on Discoveries is already drawn up by Cap. King, one of the 2 Commanders who brot the two ships home after the death of Cap. Cook, but probably we shall not see it published these 2 years, owing to the delay of the plates, which are to be executing in the very best manner by the best engravers. You will perhaps smile when I tell you I am become *Astrologer General* to the Stationers' Co having entered into agreemt with them for the sole conducting of all the Almanacs. Not that I can execute them all myself, but shall likely employ assistants to make most of the calculations & certainly all the Prognostications, these belonging to a Science far above my pitch. The conduct of the almanacs had been in the family of the Wings (24), from father to son, ever since the famous Vincent Wing Astronomer & Astrologer, & has only ceased with the death of the late Tycho Wing the last year ; the present representative of the family being a young man not at all concerned in such matters, the Co [thought] that it would be proper to make a change. Speaking of these things brings to my mind a good anecdote of a young attorney of N.C. whom we know very well which I shall here copy in the words of my correspondent who lives near the Mr. Wright in Nottinghamshire mentioned below, which Mr. Wright has for many years written the Prognostications in Moore's Almc & has the character of a great conjurer, telling the people their fortunes &c. & to this Sidropher it seems our curious townsman applied himself. My correspondent's words are, " Mr. Wright shew'd me a letter from a gentleman of Newcastle, I think a Mr. H—— an attorney, giving him an account of his second marriage & elegant situation. This gentn, some years since, sent to Mr. Wright to calculate his nativity ; and he, like all other fortune-tellers, told him great things. Afterwards having paid his addresses to two ladies at once, he sent to Mr. Wright, who had told him he was to have two wives, to know whether he should marry A or B ; he ordered him to marry A, which he did & in the letter which I saw he acquaints Mr. Wright with her death, & that he had just married B "! This may afford a laugh at N.C. sometime.

I remain,

Dr Sr Yours most sincerely,

CHA. HUTTON.

I send pr my son 2 copies of the paper on Cubic Equas to Mr. Anderson of N.C.

WOOLWICH COMN.

MY DEAR FRIEND,

We both (Mrs. H. & myself) thank you a thousand times most sincerely & affectionately for your kind remembrance of us, & especially of our dear lovely angel, for ever to be lamented. O my friend never did I think it possible I could feel such severe affliction! And sure never parents had so much cause. It is indeed impossible to express her excellence. This mournful event has quite altered all my future views & plans. I had begun a manufactory & otherwise engaged in plans of business. Now I am hastening out of them with all speed, in order to return to a retired & recluse situation for the gloomy remainder of a lingering existence.

I am sorry, my friend, on several accounts that of late years you have written so little, as your writing if your health would permit, might be so

[24] For the astronomical-astrological dynasty of the Wings, see *D.N.B.*

cheering to your friends, & so useful to the public. But since you last left us, my dear friend, many has been the heavy ream of paper I have been obliged to write. The Dictionary alone (not half my occupation) has amounted to upwards of 3 reams of folio paper, all written, & mostly by my own hand, except some copying by the dear hand of my departed angel. An immense task. I now however have reached the end of it, & I am repaid by the very favourable manner in which it is received by the public. It will add much to my happiness if it yield any amusement to my friend in the perusal, for whose sake I am sorry it is printed in so small a type, much against my will. Two parts are now published, which you receive with this, making the 1st vol. but wanting the preface, which will accompany the fourth or last part. These I order to be sent by the proprietor Mr. Johnson. Along with them I send a present of one of the Copies of the Logarithms presented to me by Mr. Baron Maseres (25), along with his Compliments to you. There is another vol. of the Logs printing.

I remain, with sincere regard & affection,

My Dear friend,

Your obliged Servant

CHA. HUTTON.

P.S. I send also 2 Copies of the pamphlets relating to the disputes in the R. Soc. one for yourself, & the other to bestow where you best may.

Mr. R. Burdon, the member for Durham, has treated me very shabbily. The project for the Stone Bridge at Sunderland he referred to the Consideration of myself & Mr. Soane (26) an eminent architect. On this business I had a vast deal of trouble, many calculations, & a great many journeys to London, & besides one's judgment may be supposed worth something. In consequence of our report & advice the Commissioners saved a great many thousand pounds. When all was over, we expected they would make us a handsome present, as the genteelest way. Instead of that Mr. B. desired to know what quantity of trouble we had had, and how much we rated it at, & would not be put off, but insisted upon our naming a certain sum. And, as if this were not degrading enough, after we had thus been brought to name 150 gs each for our Judgment & opinion, he refused to give us more than 50 for the whole!

WOOLWICH, *Aug.* 13, 1795.

MY DEAR FRIEND,

I really thought you had been possessed of my Logarithms & Conic Sections, & Tracts. I send orders now to Mr. Johnson to supply all these (none of which are my property) & to send them through Mr. Ritson along with the 3d part of the Dictionary. The two vols of Scriptores Log please to bestow any where that you please. Thank you for the list of Errata in the Dictionary, & pray be so good as to continue the collection of them.

[25] F. Maseres, "Mathematician, historian and reformer", Cursitor-Baron of the Exchequer. Although a reformer, he clung to old customs (see Lamb's *Essays of Elia*), and in mathematics he and William Frend, De Morgan's father-in-law, resolutely rejected the concept of negative numbers. Maseres is best known mathematically for his *Scriptores Logarithmici*.

[26] "Mr. Soane, an eminent architect", is possibly John (later Sir John) Soane, the architect of the Bank of England and founder of the Soane Museum. At this date Soane already had won a silver and a gold medal of the Royal Academy, and must have been known as a promising young man. Burdon is a well-known Durham surname.

I have just received from the West Indes, my son, with the loss of his right eye by a musket shot; & along with him, my grandson, the son of my daughter Camilla, who with her husband Capt. Vignoles died of the yellow fever in the West Indies; a most noble dashing boy of 2 years old. There is one of the Errata, above mentioned, I cannot make out; it is this, [p 57 l Arian]; be so good as explain it.

Mrs. Hutton is laid up in bed, with the Gout in both feet, but desires to be commended to you, along with

Your affectionate friend
and obliged servant
CHA. HUTTON.

SIR,

I have here sent you Clark, but if Mr. Hutton publishes any of my observations upon it, I desire he may publish them as his own, for I will have nothing to do with the fellow. You have seen (I doubt not) the remarks published in the Gent. Magazine for March against my Astronomy, and my answer in the last magazine. He is some such fellow as Clark. But I have a fuller solution, which I wud not publish in my own name, but get some friend to publish; after I see what answer they give to what I have written. I hear from London, that two sets of Ladies' Diary questions are publishing, and both of them give us all the trifling things, along with what is valuable. I wud advise all the Trifles to be struck out. Printing is dear, and who will buy them.

I am Sr your
most humble servt
W. EMERSON.

HURWORTH
Oct. 1. 1771.

1486. Who has forgotten the moment, if ever he experienced it, when, hovering on the outskirts of the calculus, he was enabled to guess what a mathematician is, just as he had guessed one morning what poetry was and had seen all words, as he now saw all figures and symbols, in a radiance of significant order? Whoever remembers this, though even the foothills of the calculus have since become too steep for him, remembers an expansion of his whole life, an enduring advance in his realization of himself and of the world, and is grateful to the mild little men . . . by whom these things come.—Charles Morgan in "Menander's Mirror", *Times Literary Supplement*, March 31, 1945. [Per Mr. A. P. Rollett.]

1487. William Bowes, of Yorkshire, the best bowler of his kind in cricket to day, is a very learned-looking man. I have sometimes suspected that "Bowes" is only a term he uses for cricket, and that, like Will Hay, he conducts private research under his own name, and probably lectures to a small but discerning coterie of fellow-scientists. In the lounge of an hotel, the evening before an Australian Test of 1934, he presented, for my solution, a geometrical problem over the sherry. He might as well have invited Professor Einstein to field for him at close short-leg.—*Observer*, May 11, 1941.

GEOMETRY IN PRIMARY SCHOOLS: AN INTRODUCTORY COURSE.

By R. S. Williamson.

One feature of this course may interest some readers who are not concerned with young children. While I was developing it I was interesting myself in ancient Egyptian mathematics. Whether this influenced my decision to introduce length by way of $\frac{1}{4}$ inch-squared paper, I do not know. But certain it is that, having decided thus, I at once recalled the Egyptian use of a squared surface, and it was this that led me to suggest the introduction of cartesian coordinates in a simple form to young children. It is pleasing to give this specific instance of the suggestive value of the history of mathematics to the modern teacher.

During the past twenty years or so the value of geometry for primary schools has become increasingly recognised, and the Board of Education now recommends it strongly. At present it is often treated in a scrappy way as part of the arithmetic, or merely as geometrical drawing. Ballard and Hamilton's course seems to have been the only serious attempt to develop the subject thoroughly, and the recent report, *Arithmetic in Junior Schools* (Longmans, 1940), while stressing the importance of spatial training, covers much the same ground. It seems to me, however, that the need for a thorough basis of theory needs emphasising, and, in fact, that geometry for young pupils should, in logical development, coherence and mathematical rigour, at least attain the standard of their arithmetic in these respects, and this without detriment to its concrete character and realistic applications. I take, therefore, as the general aim of a unified course, the development of a geometrical interest in the pupil's environment through a rigorous study of space in its three aspects of form, direction, and magnitude. Three standard shapes, square, oblong, circle, their halves, and, to some extent, their quarters, along with three standard slopes, 30, 45, 60 degrees (not so named), are the basis of the course. The practical work includes shape-building with geometrical tiles, drawing on $\frac{1}{4}$-inch squared and plain paper, paper-cutting and paper-folding, making simple articles, simple mensuration. The course is fully correlated with the usual arithmetic course, developing *pari passu* with it, but at the same time retaining complete geometrical independence. It is intended for pupils of mental age about 8.

Drawing and mensuration, which are essential, for both impel attention to the geometrical details of a figure, are introduced at the outset. The inch is too large a unit for useful calculations, and it has not yet been studied. But a unit of measurement is to hand on the squared paper. Each of the equal bits of a grid line are described as units, and shown to correspond to the sticks, or arithmetical units, with which the pupil builds up or counts numbers. He builds up a line, say 6 units long, with sticks, and learns to draw it along a grid line by counting the bits. He finds these units on a 9-inch ruler simply marked in quarter-inches, and learns to measure with the ruler. After a little work with lines, long and short, equal and unequal, down and across, he is ready to commence the study of *shape*.

He examines a square, side 4 units, given on $\frac{1}{4}$-inch squared paper. He draws a copy, then cuts out one, as a square tile, S, from a card of printed squares. Alternatively, the pupil may draw on squared paper, or card, the tiles for cutting out. This would be done after examining the 8-unit square. He fits the S tile on the given square in as many ways as possible. Thus, with some prompting, he finds simple properties of a square, and learns to use simple terms, *e.g.* shape, figure, side, corner. Similarly, he

examines a given square, 8 units side, divided into four equal squares, and builds it up with *S* tiles. He then uses *S* tiles, first to cover given shapes, then to build up familiar shapes, *e.g.* cross, hat, bridge, printed full size, and ½-size, on squared and plain paper. Oblong and half-square are treated similarly, with *O* and ½*S* tiles. In all this work there is good practice in calculation, based on the question, " How many units of line have I drawn? "

Simple work on *direction* follows. The pupil plots points, *e.g. A*, 6 units up from *B*, to fix the four standard directions, up, down, right, left, clearly in his mind. By joining some of these points the standard shapes already studied are formed, their properties revised and seen to be connected with the directions of the sides. The *centre*, or halfway point, of a line is found, and directions of a clock hour hand at 3, 6, 9, 12 o'clock noted. This work leads to the *circle*. The pupil is accustomed to handle coins freely, so tiles are unnecessary. After a preliminary examination of a given circle (no sides or corners ; rim), radius 4 units, he draws a circle round a penny, and then sees in how many ways the penny will fit on it. The given circle, showing centre and one centre line, is examined, to discover the constant distance of rim from centre. He constructs a circle, freehand, by joining 8 or more points, each 4 units from *O*, then cuts it out and fits it on the given circle. The *half-circle* follows, by cutting along a centre line. Revision, with a special project or two, *e.g.* drawing a flag and building it with tiles, completes the term's work.

Inch and half-inch are now introduced, with drawing on plain paper, involving the standard shapes again. The study of half-oblong, 7×4 units, for all practical purposes half an equilateral triangle, side 8 units, leads naturally to *direction*, based on three standard slopes. Two boys are pictured getting over a wall by using, respectively, ladder and plank, whose slopes, roughly 60 and 30 degrees, are compared visually. Three given drawings follow, showing lines *s*, *ff*, *t*, making angles 60, 45, 30 degrees, each with a separate ground line. The slopes are compared visually, and the figures then drawn by using coordinates (4, 7 ; 7, 7 ; 7, 4), *e.g.* " Mark *M*, 7 units right from *O*. Then *P*, 4 units up from *M*. Draw *OM*, *OP*. Mark *OP*, *s*, *ff* or *t*, to show how much it slopes. Which of your tiles fits on the corner you have made? " (*Note* : The standard slopes from 90° to 0° may be remembered by the symmetrically literal mnemonic, *n*, *sf*, *s*, *ff*, *t*, *ft*, *n*.) The pupil also draws round the edges of ½*S* or ½*O* tile, placed in a specified position, and labels sloping lines, *s*, *ff* or *t*. Thus *shape*, *direction* and *slope* are seen to be intimately connected, an important piece of work in unifying the course. Further study of *shape*, ¾-square and ¾-oblong (cf. arithmetic), leads to the *circle* again. The pupil draws a given clock face, radius 8 units, by using, in a symmetrical way, coordinates 4, 7 and 7, 4, again. He tests the shape, cuts it into quarter-circles and studies them along with quarter-hours. Revision, and Projects (making, *e.g.* postcard, luggage label) follow.

Paper-folding, based on the standard shapes, is now taken up, the pupil studying *balance*, in particular, in this way. He learns also to balance with blue tiles a shape made with red ones, thus completing a standard shape. Next, *surface*, a word introduced earlier as equivalent to face (of a tile), is studied. He finds from his squared paper, that surfaces, like lines, are made of units. Each grid square is a unit of surface, or *square unit*, just as a bit of grid line is a line unit. He returns to the *circle* and finds centre lines and centre, by paper-folding, and then studies the balance of circle, and of ½-circle. This last work is important, to complete the part played by *balance* as a unifying idea. It clinches the relation of the circle to the other standard shapes already shown through the ideas of *centre*, *shape*, and *direction*. This completes the syllabus, which is now revised fully in various ways, *e.g.* building

vital shapes (man upright, hen, cat) and ornamental patterns (cushion, bedspread) with tiles, making things (box, folded letters), drawing, etc.

One of the chief difficulties in the course is that of language. In consequence, hard technical terms have been avoided. Instead, simple or familiar words, *e.g.* level, upright, centre line, rim, square corner, sharp, blunt, cross-line, have been given a clear and definite meaning, and used freely. Missing-word questions, too, play an important part. Another difficulty, that of supervision, has been met by providing exercises suitable for mass correction, *e.g.* pupils may hold up their tiles in position to show balance.

R. S. WILLIAMSON.

PLYMOUTH AND DISTRICT BRANCH.

A GENERAL MEETING of the Branch was held at Plymouth High School for Girls on 12th February, 1946, when the Secretary's Report and Balance Sheet for the previous year were read and approved.

The President, Dr. M. Stimson, read a letter of resignation from the Secretary, Mr. F. W. Kellaway, on his leaving Plymouth to take up appointment as Principal, Apsley Day Continuation School and Evening Institute, Hemel Hempstead. It was resolved to send a letter of appreciation to Mr. Kellaway of the work he had done for the Branch since its inception, and in particular during the most difficult years of the war.

Mr. W. G. Tamlin was elected as Secretary; while the President, Vice-President and Committee were asked to remain in office for a further year.

The meeting then considered the future of the Branch and it was resolved to meet as frequently as practicable both in Plymouth and in other towns in Devon and Cornwall.

Communications for the new Secretary should be addressed to The Garth, Noss Mayo, Nr. Plymouth. It is especially requested that members not receiving notices of meetings should write to him.

W. G. TAMLIN, *Hon. Secretary.*

THE FRENCH SCIENTIFIC OFFICE.

THE FRENCH SCIENTIFIC OFFICE is to carry on and expand the work done during the past two years in this country by the French Scientific Mission. In particular, it is hoped that a small scientific library will be established at the Institut Français in the beginning of May. This will contain a number of current scientific periodicals and some recent French scientific books. The facilities offered are:

(*a*) consultation of books in the library;
(*b*) borrowing by post;
(*c*) arrangements to obtain from France, as far as possible, any books or periodicals desired which are not available in the library.

Members who wish to be enrolled on the register of the library should write to the Librarian, Institut Français, Queensberry Place, South Kensington, London, S.W. 7. They will be sent a catalogue of the books and periodicals available.

A GENERAL THEOREM IN LAPLACE TRANSFORMS.

By N. W. McLachlan.

1. *Introduction.* As usual we define the p-multiplied L.T. of $f(t)$ to be

$$\phi(p) = p\int_0^\infty e^{-pt} f(t)\,dt, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

and we adopt the abbreviation

$$f(t) = \phi(p), \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

introduced in 1938.* The theorem in § 2 was obtained by the author in 1940. It would have been published, together with other work on L.T., some years ago, but for war-time restrictions.

2. *Theorem.* If $f(t) = \phi(p)$ and $\xi(x, t) = \phi_1(p)h(p)e^{-xh(p)}$, then

$$\int_0^\infty \xi(x, t)f(x)\,dx = \phi_1(p)\phi[h(p)], \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where $\phi_1(p)$, $h(p)$ are continuous functions of p, independent of x, $R[h(p)] \geqslant p_0 > 0$, x real and $\geqslant 0$.

Proof. Writing $h(p)$ for p in (1) § 1, we get

$$\phi[h(p)] = h(p)\int_0^\infty e^{-th(p)} f(t)\,dt. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

By hypothesis $f(t)$ has the L.T. $\phi(p)$, so $\int_0^\infty e^{-pt} f(t)\,dt$ converges for $R(p) \geqslant p_0 > 0$, and, therefore, (2) converges if $R[h(p)] \geqslant p_0 > 0$. Multiplying both sides of (2) by $\phi_1(p)$ and replacing t by x leads to

$$\phi_1(p)\phi[h(p)] = \int_0^\infty \phi_1(p)h(p)e^{-xh(p)} f(x)\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

By hypothesis

$$p\int_0^\infty e^{-pt}\xi(x, t)\,dt = \phi_1(p)h(p)e^{-xh(p)}, \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

the upper limit of the integral being $+\infty$, provided the r.h.s. of (4) does not have e^{-px} as a factor. This point is explained below, (6).

Substituting from (4) into (3) gives

$$\phi_1(p)\phi[h(p)] = p\int_0^\infty e^{-pt}\left[\int_0^\infty \xi(x, t)f(x)\,dx\right]dt, \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

provided that the order of integration may be inverted. Then it follows from (5) that

$$\int_0^\infty \xi(x, t)f(x)\,dx = \phi_1(p)\phi[h(p)]. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

If $g(t) = p\,\phi_1(p)$, by the "shift" theorem we get

$$g(t - x) = e^{-px}\,p\,\phi_1(p).$$

Thus if $h(p) = p$, the r.h.s. of (4) becomes $e^{-px}\,p\,\phi_1(p)$ and $\xi(x, t)$ assumes one of the forms $\xi(t - x)$, $\xi(x, t - x)$. Moreover, the function ξ is considered in the range $t > x \geqslant 0$, so the upper limit is now t and the lower limit $x \geqslant 0$.

* McLachlan, *Phil. Mag.*, 26, 394, 1938. The symbol has been adopted by French engineers and mathematicians.

3. *Conditions for inverting the order of integration in* (5) § 2. If the shift factor e^{-px} is absent from the r.h.s. of 4 § 2, there are two sets of conditions. Each set is sufficient, but certain members are not always necessary. The order of integration may be inverted if

1°. (*a*) $f(x)$ is continuous in $x \geqslant 0$,

(*b*) $\xi(x, t)$ is continuous in $x \geqslant 0$, and in $t \geqslant 0$,

(*c*) $\int_0^\infty e^{-pt}\xi(x, t)f(x)dx$ is uniformly convergent in $t \geqslant 0$,

(*d*) either $\int_0^\infty e^{-pt}dt\int_0^\infty \xi(x, t)f(x)dx$ or $\int_0^\infty f(x)dx\int_0^\infty e^{-pt}\xi(x, t)dt$ is absolutely convergent.

2°. As at (*a*), (*b*) in 1°.

(*c*) As in 1° (*c*), but in the range $0 \leqslant t \leqslant h$.

(*d*) $\int_0^\infty f(x)dx\int_0^t e^{-p\tau}\xi(x, \tau)d\tau$ is uniformly convergent in $t \geqslant 0$.

3°. When the r.h.s. of (4) § 2 contains e^{-px}, the conditions are :

(*a*) $f(x)$ is continuous in $0 \leqslant x \leqslant t$,

(*b*) $\xi(t-x)$ or $\xi(x, t-x)$ is continuous in $0 \leqslant x \leqslant t$,

(*c*) $\int_x^\infty e^{-pt}\left\{\begin{matrix}\xi(t-x)\\ \xi(x, t-x)\end{matrix}\right\}f(x)dt$ is uniformly convergent with respect to x in $0 \leqslant x \leqslant t$.

4. *Particular cases.*

(*a*) When $\phi_1(p) = 1$, (6) § 2 degenerates to

$$\int_0^\infty \xi(x, t)f(x)dx = \phi[h(p)]. \quad \ldots\ldots(1)$$

(*b*) When $h(p) = p$, (1) takes the form

$$\int_0^t \xi(t-x)f(x)dx = \phi(p), \quad \ldots\ldots(2)$$

the upper limit in the integral being t, in accordance with the remark below (6) § 2. Thus

$$\int_0^t \xi(t-x)f(x)dx = f(t), \quad \ldots\ldots(3)$$

provided $f(x)$ is single-valued and continuous at $x = t$. Here it can be shown that $\xi(t-x)$ is the impulsive function $I(t)$ at $x = t$, where $I(t) = p$.*

(*c*) Writing $\phi_1(p)/p$ for $\phi_1(p)$, and $h(p) = p$ in (4) § 2, leads to

$$\xi(t-x) = e^{-px}\phi_1(p), \quad \ldots\ldots(4)$$

so

$$\xi(t) = \phi_1(p). \quad \ldots\ldots(5)$$

Substituting in (6) § 2, and applying the paragraph beneath the latter, we obtain

$$\int_0^t \xi(t-x)f(x)dx = \phi_1(p)\phi(p)/p, \quad \ldots\ldots(6)$$

which is the well-known " product " theorem.

* *Phil. Mag.*, **26**, 695, 1938, and *Math. Gaz.*, **23**, 270, 1939.

5. *Applications.* (1) § 2 may be used to derive a number of theorems given on p. 12 of *Formulaire pour le Calcul Symbolique.**

(*a*) Show that

$$\int_0^t J_0[2\sqrt{\{x(t-x)\}}]f(x)dx = [p/(p^2+1)]\,\phi[(p^2+1)/p].$$

In (1) § 2 take $h(p)=(p^2+1)/p$, $\phi_1(p)=p/(p^2+1)$, and (4) § 2 gives

$$p\int_x^\infty e^{-pt}\xi(x,t)dt = e^{-px}e^{-x/p}. \quad \text{.....(1).}$$

By formula 7, p. 28 of the above list,

$$J_0\{2\sqrt{(xt)}\} = e^{-x/p}, \quad \text{.....(2)}$$

so

$$\xi(x,t) = J_0[2\sqrt{\{x(t-x)\}}], \quad \text{.....(3)}$$

$t>x$, since (1) has the factor e^{-px}. Hence by (6) § 2, we get

$$\int_0^t J_0[2\sqrt{\{x(t-x)\}}]f(x)dx = [p/(p^2+1)]\,\phi[(p^2+1)/p]. \quad \text{.....(4)}$$

In (3), $\xi(x,t)$ has the form $\xi(x,t-x)$, and condition 3° (*b*) is satisfied. Hence for (4) to be true, $f(x)$ must be such that conditions 3° (*a*), (*c*) are satisfied also.

If in (4), (p^2-1) be written for (p^2+1), $R(p)>1$, J_0 must be replaced by the modified Bessel function I_0.

(*b*) Show that

$$(1/2^n t^{\frac{1}{2}(n+1)}\sqrt{\pi})\int_0^\infty e^{-x^2/4t}He_n(x/2\sqrt{t})f(x)dx = p^{\frac{1}{2}n}\phi(p^{\frac{1}{2}}),$$

where He_n is the Hermite polynomial of order n.

In (4) § 2, let $\phi_1(p)=p^{\frac{1}{2}n}$, and $h(p)=p^{\frac{1}{2}}$, then

$$\xi(x,t) = p^{\frac{1}{4}(n+1)}e^{-xp^{\frac{1}{2}}} \quad \text{.....(1)}$$

From formula 3 on p. 55 of the list,

$$\xi(x,t) = e^{-x^2/4t}He_n(x/2\sqrt{t})/2^n t^{\frac{1}{2}(n+1)}\sqrt{\pi}, \quad \text{.....(2)}$$

so the above result follows immediately, provided $f(x)$ is such that one of the sets of conditions in § 3 is satisfied. If $n=0$, we have the particular case

$$\frac{1}{\sqrt{(\pi t)}}\int_0^\infty e^{-x^2/4t}f(x)dx = \phi(p^{\frac{1}{2}}) \quad \text{.....(3)}$$

(*c*) By writing

$$\phi_1(p)=p/(p^2-a^2),\ h(p)=(p^2-a^2)^{1/2}$$

in § 2, we derive the theorem

$$\int_0^t f(w)I_0[a(t^2-w^2)^{1/2}]dw = [p/(p^2-a^2)]\,\phi[(p^2-a^2)^{1/2}] \quad \text{.....(4)}$$

which is useful in solving technical problems.

N. W. McL.

* By N. W. McLachlan and P. Humbert. See *Math. Gaz.*, **29**, 241, 1945.

MATHEMATICAL NOTES.

1887. *The "continuous" unit matrix.*

The infinite unit matrix I is defined by $i_{n,k}=1$ for $n=k$, $i_{n,k}=0$ otherwise, n and k taking positive integral values only.

The infinite integral $I(x,y)=\int_0^\infty (u^t \log_e u - a^t \log_e a)\,dt$, where

$$u(x,y)=\frac{2(x+b)(y+b)}{(x+b)^2+(y+b)^2},$$

a and b being constants such that $0<a<1$, $b>0$, defines the function $I(x,y)$ for every $x\geqslant 0$, $y\geqslant 0$, and there

$$I(x,y)=\Big[u^t-a^t\Big]_{t=0}^{\infty}.$$

Now for $x=y$, $u=1$; hence $I(x,y)=1$,

and for $x\neq y$, $0<u<1$; hence $I(x,y)=0$,

thus $I(x,y)$ defines the "continuous" unit matrix independently of the parameters a and b.

More generally, if $v(x,y)$ and $w(x,y)$ are real functions of the real variables x and y, and we define

$$u=2vw/(v^2+w^2), \text{ for } v^2+w^2\neq 0,$$
$$u=1, \text{ for } v=w=0,$$

then $I(x,y)=1$ on the curve $v(x,y)=w(x,y)$, and $I(x,y)=0$ everywhere else.

For example, if $v(x,y)\equiv x^2+y^2$, $w(x,y)\equiv b^2$, then

$$u(x,y)=2b^2(x^2+y^2)/(x^4+2x^2y^2+y^4+b^4),$$

and $I(x,y)=1$ on the circle $x^2+y^2=b^2$, and $I(x,y)=0$ everywhere else.

P. VERMES.

1888. *Faulty mathematics.*

In the recent *Report on School Certificate Mathematics*, a Specimen Examination Paper is given on page 9, and in it Question 2 (i) is: "If $A=\frac{1}{2}h(b+l)$, find l when $A=50$, $b=3$ and $h=12$."

Questions similar to this (usually not so easy) have appeared in many papers set by different examining bodies during the past twenty years or so—but with one unfortunate addition—the candidate has almost always been required to rearrange the formula ("to change its subject") so that the quantity of which the numerical value is to be found stands as its "subject".

Naturally teachers have coached their classes in answering such a common type of question; in consequence a number of their former pupils must believe that in using a formula like that in the question above, in order to find l, say, the proper procedure is, first, to "change the subject" to l, and then to substitute the given numerical values. This, of course, is a waste of time, because it is quicker to substitute in the original formula and then to solve the resulting equation; it is not only a waste of time, it increases the risk of error, because the manipulation of literal quantities is more tedious than that of numerical quantities, and of course if an error is made in rearranging the formula, then although the subsequent arithmetical work is correct, the answer will be wrong.

JAS. W. STEWART.

1889. *Evaluation of π. Are Shanks' figures correct?*

There are probably a good many people who know that the value of π has been calculated to 707 decimal places, though not so many know the name of the man who carried out the calculation; but it is probably not so widely known that Shanks produced three separate values—the first to 530 places in 1853, the second to 607 places in the same year, and finally to 707 places in 1873. To the best of my belief these figures have not so far been checked beyond the 500th place; the *Encyclopaedia Britannica* (from which I extracted the information given above) gives a list of names of the men who have calculated the value of π, and the only name given for a result beyond the 500th place is that of Shanks himself.

In May, 1944, a colleague of mine at this college, Mr. R. W. Morris, produced a series for the evaluation of π which we believe to be original, namely

$$\frac{\pi}{4} = 3\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{20} + \tan^{-1}\frac{1}{1985}.$$

I was sufficiently interested to test the series for the first 100 places, and finding the work unexpectedly interesting, continued the evaluation (checking against Shanks' figures as I went along) until a year later, in May, 1945, I had reached the 530th decimal place—*the very place which Shanks reached in his first published result.* Up to this point, whenever I had disagreed with Shanks' figures (and this has occurred from time to time, owing to copying errors, etc.), I had never had any real difficulty in finding where I had gone wrong. But at this point I not only found my figures differing completely from those of Shanks, but all my efforts to find my mistake failed. After three weeks or so of fruitless effort, I eventually combined two terms of the series $(\tan^{-1}\frac{1}{20} + \tan^{-1}\frac{1}{1985})$ into one $(\tan^{-1}\frac{5}{99})$ and spent most of my spare time in the next four months carrying out this independent check, and was eventually rewarded by *completely confirming the accuracy of that part of my calculation.*

I could not find a satisfactory independent check for $\tan^{-1}(\frac{1}{4})$, so I tightened up the checks I had already been using, put in yet another one which I believe eliminates the possibility of a *copying error*, and still could find no mistake. I have shown my checking system to three or four of my colleagues, and they all agree that there are so many checks and counter-checks that the possibility of an error seems to be negligible.

I should of course be delighted if any one would care to check my working, and equally pleased if any one could tell me how I can get hold of the details of Shanks' working, so that I could check his work and find the mistake which I believe he must have made.

If, in spite of all my precautions, I am still wrong, it is, to say the least of it, a curious coincidence that it should have occurred at the very place where Shanks first stopped, and beyond which, as I have said, nobody but Shanks has ever yet gone.

Any information or suggestions relative to this matter will be gratefully received.

In case any one is interested, I give the figures from the 521st place to the 540th place (i) as Shanks gave them, (ii) as I think they should be:

(i) 86021 39501 60924 48077 (Shanks),

(ii) 86021 39494 63952 24737 (D. F. F.).

In calculations made since the preceding part of this Note was written,

from the 569th decimal place onwards I have a further disagreement with Shanks' figures, namely

68965517241379310344827586 20 (recurring).

These are *precisely* the figures which should occur at this *exact* spot (i.e. from the 569th d.p. onwards) in the term $\frac{(1/5)^{145}}{145}$ which comes in the series for $\tan^{-1}(1/5)$. This surely cannot be a mere coincidence.

It looks very much as if Shanks omitted the whole of this term from the 569th place onwards. D. F. FERGUSON.

1890. *Computation of* $n!$

Lindelöf* gives the summation formula

$$\sum_a^n \phi(r) = \int_{a-\frac{1}{2}}^{n+\frac{1}{2}} \phi(x)dx - \tfrac{1}{24}[\phi'(n+\tfrac{1}{2}) - \phi'(a-\tfrac{1}{2})] + \tfrac{7}{5760}[\phi'''(n+\tfrac{1}{2}) - \phi'''(a-\tfrac{1}{2})] - \dots, \quad \dots\dots\dots\

which is correct to 6 significant figures, while if the correction terms in $1/n^3$, $1/n^5$ are taken into account,

$$\log_{10} 10! = 6{\cdot}55976299,$$

with an error of less than one in 100 million.

Incidentally, in working with large n the expression $(n+\frac{1}{2})\{\log(n+\frac{1}{2}) - \log e\}$ is easier to handle than the expression $(n+\frac{1}{2})\log n - n\log e$ in Stirling's formula.

J. GORDON SKELLAM.

1891. *Triangle properties.*

In continuation of Note 1715, referring to Fig. 1 in which

$$BA_1 : A_1C = CB_1 : B_1A = AC_1 : C_1B = m : n,$$

I should like to add the following results of a, to me, very interesting investigation.

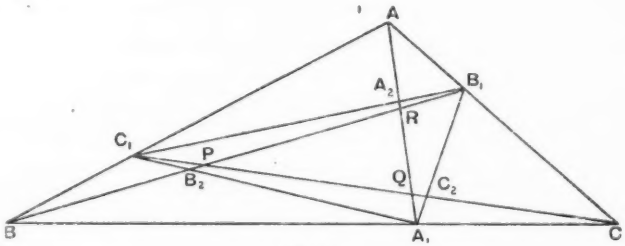


FIG. 1.

(i) The triangle with sides equal respectively to AA_1, BB_1, CC_1 has a Brocard angle equal to that of the triangle ABC.

(ii) The area of this triangle (AA_1, BB_1, CC_1) is unaltered by interchange of m and n, being equal to that of any of the quadrilaterals such as BCB_1C_1.

(iii) The sides of the triangle $A_1B_1C_1$ are divided at A_2, B_2, C_2, so that

$$B_1A_2 : A_2C_1 = C_1B_2 : B_2A_1 = A_1C_2 : C_2B_1 = n^2 : m^2,$$

and therefore the "equal Brocard angle" property extends also to the triangles $A_2B_2C_2$, $A_3B_3C_3$, and so on.

The ratio of the areas of the triangles $A_rB_rC_r$, ABC is

$$\{(m^3+n^3)(m^6+n^6)\ldots\} : \{(m+n)(m^2+n^2)\ldots\}^3,$$

where in the first bracket the indices run up to $3 \cdot 2^{r-1}$ and in the second to 2^{r-1}.

(iv) The shape of a triangle is unique if its Brocard angle Ω is 30°, for it can be shown that

$$\cos 2\Omega = \tfrac{1}{2}\left\{1 + \frac{\Sigma(a^2-b^2)^2}{2\Sigma a^2b^2}\right\},$$

and therefore when $\Omega = 30°$ it follows that $\Sigma(a^2-b^2)^2 = 0$, so that $a=b=c$. It follows also that for all other triangles $\Omega < 30°$.

(v) If ABC be a light framework on a smooth horizontal plane, and if

equal masses (say, 3 flies) move on the rough upper surface, from B to C with velocity ka, from C to A with velocity kb, and from A to B with velocity kc,

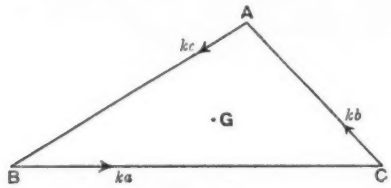


FIG. 2.

where k is a constant, starting simultaneously, then if $d\theta/dt$ be the angular velocity of the framework about its centroid G (clearly at rest), then

$$d\theta/dt = \sqrt{3}k \tan \Omega/2(1 - 3kt + 3k^2t^2),$$

whence $$\theta = \sqrt{3} \tan \Omega \,.\, \text{arc tan } \{kt\sqrt{3}/(2 - 3kt)\}.$$

When $kt = 1$, each mass has traversed a side of the triangle and $\theta = \sqrt{3} \tan \Omega \,.\, \frac{1}{3}\pi$, which has a maximum value, $2\pi/3$, when the triangle is equilateral.

This dynamical appearance of the Brocard angle might perhaps be the basis of a good scholarship question. The special case of an equilateral triangle is interesting, for here the angular velocity, relative to the framework, of the line joining one of the masses to G will be found to be equal and opposite to that of the framework relative to the plane; thus the whole motion is in a way pulsating, each fly journeying in a straight line halfway to G and back again to its old position but new vertex. J. STORR-BEST.

1892. *How not to state a theorem.*

In C. A. Laisant's *Géométrie du Triangle* (1896, Nos. 18–19) is quoted a theorem: "Soient O le centre du cercle circonscrit au triangle ABC, G le centre de gravité, H le point de concours des hauteurs; D un point quelconque, H' le milieu de DH, G' le point divisant la droite GD dans le rapport 1 : 3. Démontrer que les points O, G', H' sont en ligne droite et que $OG' = G'H'$" (Brocard, from *Mathesis*).

A figure for this statement, correctly locating O, G and H, is not without complexity, and if drawn merely serves to emphasise that its only property which has any bearing on the issue lies in $OG = \frac{1}{3}GH$. In short, the theorem has nothing whatever to do with triangle ABC, and O, G, H *per se*, and amounts to no more than: In *any* triangle ODH, if G divides OH in the ratio 1 : 2 and H' divides DH in the ratio 1 : 1, then OH' divides GD at G' in the ratio 1 : 3 and is itself bisected at G'.

This is really a property of a complete quadrilateral rather than of a triangle, and would seem to be best put in the form (see figure opposite):

Given two of the ratios of the segments m/n, p/q, u/v, x/y, along the sides of a complete quadrilateral, find the other two.

Allowing for "symmetry", there are four cases to consider which, by comparison of areas just as in the numerical case above, give:

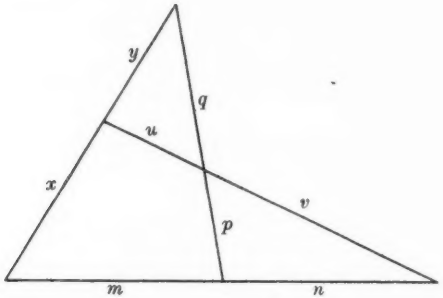
given m/n, x/y: $\quad p/q = nx/(m+n)y, \qquad u/v = my/n(x+y);$

given m/n, u/v: $\quad p/q = (mv - nu)/(m+n)u, \quad x/y = (mv - nu)/nu;$

given m/n, p/q : $u/v = mq/\{(m+n)p+nq\}$, $x/y = (m+n)p/nq$;

given p/q, u/v : $m/n = (p+q)u/(qv-pu)$, $x/y = p(u+v)/(qv-pu)$.

Examination of a considerable number of books on elementary and on "modern" geometry and trigonometry has revealed a large number of particular numerical cases, but no statement of the general results, which is somewhat surprising since there is a most extensive display of other metrics of the quadrilateral.



The results lend themselves to much more interesting special cases than the fifty or so discovered. For example :

"Given any triangle ABC and a median CF, AN_1 bisects CF meeting BC at N_1, BN_2 bisects AN_1 meeting CA at N_2, CN_3 bisects BN_2 meeting AB at N_3 and so on for AN_4, BN_5, ... The bisecting points are O_1, O_2, ... Establish the ratios made by O_n and N_n." ·

The law of formation of the ratios is easily observed in using the appropriate formulae from above, and O_1, O_2, O_3, ... divide the transversals in the series 3/1, 5/1, 8/2, 13/3, 21/5, ... ; while N_1, N_2, ... divide the sides in the ratios 2/1, 3/2, 5/3, 8/5, 13/8, ... In both these series the numerators and denominators are recurrent, each term being the sum of the two preceding. To solve this by area comparisons at, say, about O_6, N_6 is a somewhat dizzy task.

T. R. DAWSON.

1893. *Particles and cells.*

1. In statistical mechanics we deal with the distribution of (similar) particles in cells (or compartments) in the six-dimensional phase-space. Each cell has a volume h^3, h being the Planck constant : it is not necessary to refer here to the alternative version in terms of wave-functions. In quantum statistics similar particles, say electrons, are treated as indistinguishable from each other, whereas in classical statistics they are assumed to be distinguishable. The cells in every case are considered as distinguishable from each other. However, if we do not limit ourselves to the cases that occur in statistical mechanics, the following four cases arise :

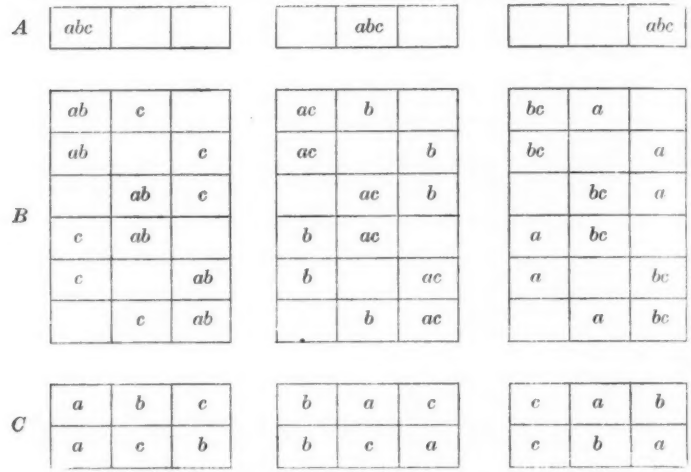
(i) Particles distinguishable from each other and cells distinguishable from each other.

(ii) Particles indistinguishable from each other and cells distinguishable from each other.

(iii) Particles distinguishable from each other and cells indistinguishable from each other.

(iv) Particles indistinguishable from each other and cells indistinguishable from each other.

As an illustration take the case of three particles (a, b, c) and three cells (I, II, III). All the possible arrangements are shown in the figure.



For case (i) all the above arrangements count as different, and the total number of complexions (arrangements) is $3^3 = 27$. For case (ii) the three arrangements of type A count as different, the six rows of type B represent six arrangements and the two rows of C give one arrangement. The total number of complexions is therefore reduced to 10. In case (iii) the three arrangements of type A count as identical, three columns of type B represent three arrangements and the six arrangements of type C denote only one arrangement, and hence the total number of complexions is 5. For the last case when the particles as well as the cells are indistinguishable, the arrangements of type A become indistinguishable from each other, the same holds for B and C and the number of complexions is reduced to 3.

2. If n denotes the number of particles, and r the number of cells, and c the number of complexions, then, assuming no restriction regarding the number of particles that a cell may contain, we have for the first two cases:

$$\text{(i)} \quad c = r^n,$$

and

$$\text{(ii)} \quad c = {}^rH_n = \frac{(r+n-1)!}{n!\,(r-1)!}.$$

For the case (iii), when the particles are distinguishable and the cells indistinguishable, the expression for c is somewhat complicated. Suppose n_i to be the number of particles in the ith cell, then any particular distribution of the n particles in the r cells may be represented as $n = k_1 n_1 + k_2 n_2 + \ldots + k_l n_l$, where k_i denotes the number of cells each of which contains n_i particles:

$n_1, n_2, \dots, n_l$ are all different. The numbers of complexions is then given by *

$$c = \Sigma n! \,/\, (n_1!)^{k_1}(n_2!)^{k_2}\dots(n_l!)^{k_l} k_1! k_2! \dots k_l!,$$

where the summation extends over all n_i satisfying

$$\sum_{i=1} k_i n_i = n, \quad \sum_{i=1}^{l} k_i = r.$$

For example, when $n=3$ and $r=3$,

$$c=5,$$

and when $n=6$ and $r=3$,

$$c=122.$$

For the case of indistinguishable particles and indistinguishable cells the number of complexions is

$$c = \sum_{k=1}^{r} p_k(n) = p_r(n+r) \quad \text{if} \quad r \leqslant n$$

and

$$= p(n) \quad \text{if} \quad r \geqslant n,$$

where $p_k(n)$ represents the number of partitions of n into k summands and $p(n)$ the number of unrestricted partitions of n.

Case (i) corresponds to classical statistics and (ii) to Bose-Einstein statistics.

3. So far we have placed no restriction as to the number of particles that a cell may contain, but if it be assumed that a cell cannot contain more than one particle, we have for the various cases :

(i) $c = \dfrac{r!}{(r-n)!}$,

(ii) $c = \binom{r}{n} = \dfrac{r!}{n!(r-n)!}$,

(iii) $c=1$,

(iv) $c=1$.

Case (ii) corresponds to Fermi-Dirac statistics.

F. C. AULUCK and D. S. KOTHARI.

1894. *Applications of the power of a point.*

An interesting series of results may be derived from the theorem that the difference of the powers of a point P with respect to two circles of centres A and B is $2AB \,.\, PN$, where N is the foot of the perpendicular from P to the radical axis of the two circles, with the corollary that if P is on one of the circles, its power with respect to the other circle is $2AB \,.\, PN$.

1. *Salmon's theorem.*

Let P, Q be any two points, X the foot of the perpendicular from P to the polar of Q, Y the foot of the perpendicular from Q to the polar of P, the

* This result may also be expressed as

$$\sum_{k=1}^{n} D_k{}^n,$$

where $$D_k{}^n = \left\{ k^n - \binom{k}{1}(k-1)^n + \binom{k}{2}(k-2)^n \dots + (-1)^{k-1}\binom{k}{k-1} \right\} \Big/ k^n.$$

(Chrystal, *Algebra*, Part II, p. 24, 1931.)

polars being with respect to a circle centre A and radius r. Draw the circle on AQ as diameter, having its centre at B. Then the polar of Q is the radical axis of these two circles. If $\angle PAQ = \theta$, then

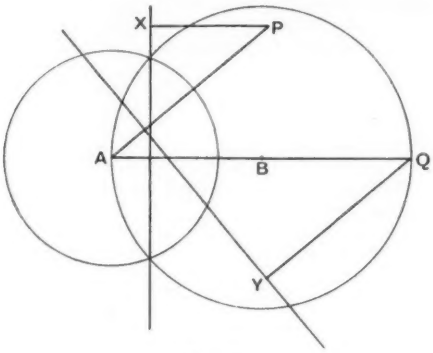


FIG. 1.

$$2AB \,.\, PX = AQ \,.\, PX = (PA^2 - r^2) - (PB^2 - AB^2)$$
$$= 2AB \,.\, AP \cos\theta - r^2 \quad \text{(from triangle } APB\text{)}$$
$$= AP \,.\, AQ \cos\theta - r^2.$$

Similarly $\quad AP \,.\, QY = AP \,.\, AQ \cos\theta - r^2.$

Hence $\quad AP : AQ = PX : QY.$

2. Let O be the circumcentre of the triangle ABC, AXP the bisector of the angle at A, CY the perpendicular to this bisector.

Then $AY = AC$ and $PY = PC$, from the kite $AYPC$. Thus

$$PB = PY = PI = PC,$$

and BC is the radical axis of the circle ABC and the circle centre P.

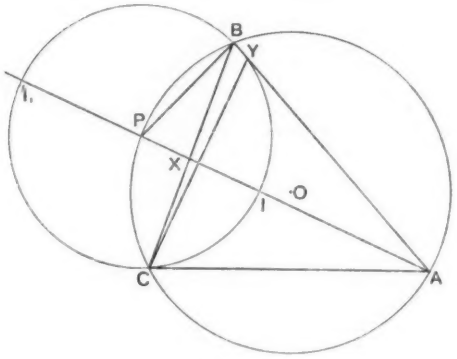


FIG. 2.

Thus equating the numerical values of the power of I with respect to the circle ABC, we have

$$R^2 - OI^2 = 2Rr.$$

3. Considering the escribed centre I_1, we have

$$OI_1^2 - R^2 = 2Rr_1.$$

4. The power of A with respect to the circle centre P is

$$AY \cdot AB = 2R \cdot AD,$$

where AD is perpendicular to BC,

that is,

$$AB \cdot AC = 2R \cdot AD.$$

5. Let O' be the image of O in BC. Then H will lie on the circle centre O' and radius R, while BC is the radical axis of this circle and the circumcircle of the triangle ABC.

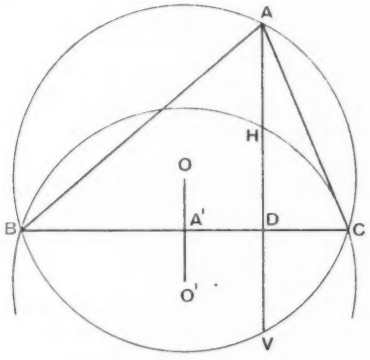


FIG. 3.

Now $OO' = 2OA' = 2R \cos A$, and $HD = 2R \cos B \cos C$. The power of H with respect to the circumcircle is

$$R^2 - OH^2 = 2OO' \cdot HD.$$

Hence $OH^2 = R^2(1 - 8 \cos A \cos B \cos C)$.

Note that if AH meets the circumcircle in V, $AH \cdot HV = R^2 - OH^2$ and $HV = 2HD$. Thus $R^2 - OH^2 = 2AH \cdot HD = 2OO' \cdot HD$, as before.

6. If G is the centroid of the triangle ABC, the power of any vertex with respect to the circle through G and the other two vertices is the same for each vertex. (Fig. 4).

For BC is the radical axis of the circles ABC, BGC. Let GK be the perpendicular from G to BC and let S be the centre of the circle BGC. Then the power of A with respect to the circle BGC is

$$2OS \cdot AD = 6OS \cdot GK,$$

which is three times the power of G with respect to the circle ABC and is independent of the chosen vertex.

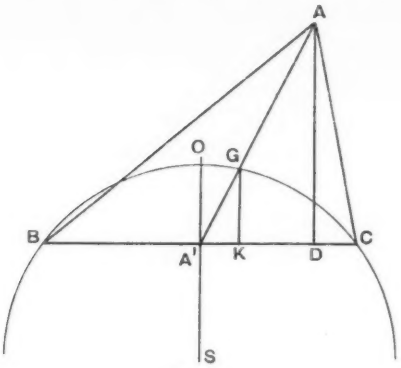


FIG. 4.

7. If P is a variable point on a fixed circle of centre A and radius a, let a circle centre P and constant radius R be drawn to cut a second fixed circle, centre B and radius b, in L, M. Then the locus of X, the foot of the perpendicular from A to LM, is a circle.

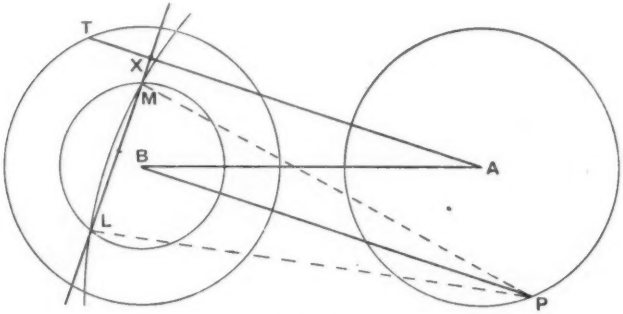


FIG. 5.

Draw the circle centre B and radius a, and let AX meet this circle in T; XA and BP are parallel, since each is perpendicular to LM, and clearly $AT = BP$. LM is the radical axis of the circle centre P and the circle centre B and radius b. Thus considering the powers of A with respect to these circles,

$$2BP \,.\, AX = (AB^2 - b^2) - (a^2 - R^2) = \text{constant},$$

that is, $AT \,.\, AX$ is constant. Now the locus of T is the circle centre B and radius a. Hence the locus of X is a circle.

8. *Use of the power in establishing a well-known theorem.*

(*a*) Internal case, referring to Figure 2.

Since $\angle PBC = \frac{1}{2}A = \angle PAB$, $PB^2 = PX \,.\, PA$, and X, A are conjugate

points for the circle centre P. Thus AX^2 is the algebraic sum of the powers of A and X; that is,

$$AX^2 = AY \cdot AB - BX \cdot XC$$
$$= AC \cdot AB - BX \cdot XC.$$

(*b*) External case. $PB = PC = PY$, and

$$\angle PBA = \angle PBC - \angle ABC = \angle PAB - \angle ABC = \angle AXB.$$

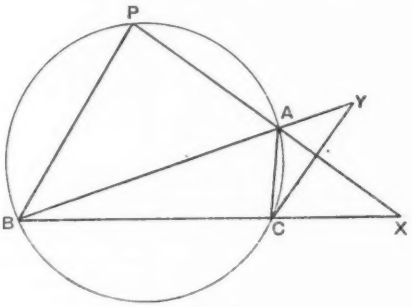


FIG. 6.

Thus $PB^2 = PA \cdot PX$ and A and X are conjugate points for the circle BCY, centre P. Hence

$$AX^2 = XC \cdot XB - AY \cdot AB$$
$$= XC \cdot XB - AC \cdot AB.$$

A. H. MILLER.

1895. *On the end digits of numbers.*

1. *Introduction.*

Let $(\ldots \alpha)$ denote any number whose last digit is α. Then it is easily seen that $(\ldots \alpha)^n = (\ldots \alpha)$ in the following cases:

1.1. n any integer and $\alpha \equiv 0, 1 \pmod 5$;
1.2. n odd, $\alpha \equiv 4 \pmod 5$;
1.3. $n = 4p + 1$, $\alpha \equiv 2, 3 \pmod 5$.

Naturally, the question arises: "Is it possible to find the values of n for which an integer x and its nth power have their last r digits the same?"

When the scale of notation p is a prime, the solution is particularly simple; this is discussed in § 2, and the extension to scale 10 is taken up in § 3.

2. *Prime base p.*

2.1. Notation.

(*a*) α denotes any number such that $0 < \alpha < p$.

(*b*) Any number x is of the form $\alpha_k \alpha_{k-1} \ldots \alpha_1 \alpha_0$, that is,

$$\alpha_k p^k + \alpha_{k-1} p^{k-1} + \ldots + \alpha_1 p + \alpha_0,$$

and x is prime to p unless otherwise stated.

(*c*) For compactness, we write where convenient $\{q\}$ for x^q.

(*d*) n_r denotes an integer such that for all x,

$$\{n_r\} - x \equiv 0 \pmod{p^r}.$$

(e) $\phi(p)$ is the number of integers less than and prime to p, and $\phi_1(p)$ is $\phi(p)$ or a divisor of $\phi(p)$.

With these notations we can prove the following results for x prime to p.

2.2. $n_1 = \lambda\phi_1(p) + 1$, where λ is any integer.

For $\{\phi_1(p)\} - 1$ is a factor of $\{\lambda\phi_1(p)\} - 1$, and $\{\phi_1(p)\} - 1 \equiv 0 \pmod{p}$.*

2.3. $n_r = \lambda p^{r-1}\phi_1(p) + 1$.

We first establish a recurrence formula for $N_r = n_r - 1$.

2.31. Assume $\{N_{r-1}\} \equiv 0 \pmod{p^{r-1}}$.

Since $\{N_{r-1}\} - 1$ is a factor of $\{kN_{r-1}\} - 1$, we have

$$\sum_{q=1}^{p-1} (\{qN_{r-1}\} - 1) \equiv 0 \pmod{p^{r-1}};$$

that is, $$1 + \{N_{r-1}\} + \{2N_{r-1}\} + \ldots + \{(p-1)N_{r-1}\} \equiv 0 \bmod (p).$$

Using 2.31, $$\{pN_{r-1}\} - 1 \equiv 0 \pmod{p^r}.$$

Hence $$N_r = pN_{r-1}.\dagger$$

By 2.2, $$N_1 = \lambda\phi_1(p).$$

Hence $$n_r = N_r + 1 = \lambda p^{r-1}\phi_1(p) + 1.$$

2.4. The case when x is not prime to p is easily solved, for x is a multiple of p and $\{n_r\}$ will contain p to the power n_r, and hence the last $(n_r - 1)$ digits are zeros. Thus $\{n_r\}$ and x will have the last r digits zero if x is of the form λp^r.

3. *Base* 10.

It is obvious that if $\{n_r\}$ and x are to have their last r digits the same, n_r must be chosen from the set $4\lambda + 1$ corresponding to n_1.

All the symbols n, x, λ, k, etc., denote positive integers and x is prime to 10 unless otherwise stated. We do not consider numbers congruent to 1 (mod 10^r), since any power of such a number is also congruent to 1 (mod 10^r).

3.1. $$n_r = 4 \cdot 5^{r-1} + 1.$$

From Fermat's theorem, $x^4 - 1 \equiv 0 \pmod{5}$, but x^4 is always of the form $4m + 1$; hence:

3.11. $$x^4 - 1 \equiv 0 \pmod{20},$$

and $$(x^4 - 1) + (x^{2\cdot4} - 1) + (x^{3\cdot4} - 1) + (x^{4\cdot4} - 1) \equiv 0 \pmod{20}.$$

Thus $$1 + x^4 + x^8 + x^{12} + x^{16} \equiv 0 \pmod{5},$$

and, using 3.11,

3.12. $$x^{20} - 1 \equiv 0 \pmod{10^2},$$

or $$x^{21} - x \equiv 0 \pmod{10^2},$$

that is, $$n_2 = 4 \cdot 5 + 1.$$

3.13. Since $x^4 - 1$ is of the form $m \cdot 2^k$, where m is odd,

$$x^4 - 1 \equiv 0 \pmod{2^k}, \text{ with } k \geqslant 4.$$

Using 3.12, $$x^{20} - 1 \equiv 0 \pmod{2^k \cdot 5^2},$$

and $$(x^{20} - 1) + (x^{2\cdot20} - 1) + (x^{3\cdot20} - 1) + (x^{4\cdot20} - 1) \equiv 0 \pmod{2^k 5^2},$$

that is, $$1 + x^{20} + x^{40} + x^{60} + x^{80} \equiv 0 \pmod{5},$$

* Barnard and Child, *Higher Algebra*, p. 437.

† This is sufficient.

whence from two lines above :

$$x^{5.20} - 1 \equiv 0 \pmod{2^k \cdot 5^3}$$

and $$x^{5.20+1} - x \equiv 0 \pmod{2^k \cdot 5^3};$$

that is, $$n_3 = 4 \cdot 5^2 + 1.$$

Similarly $$n_4 = 4 \cdot 5^3 + 1.$$

In determining n_5 a repetition of the above process gives, if $k \geqslant 5$,

$$\{5N_4 + 1\} - x \equiv 0 \pmod{10^5},$$

where $N_4 = n_4 - 1$. Using 3.13 and denoting by h the highest power of 2 contained in $x^4 - 1$, we have

$$n_5 = 4 \cdot 5^4 + 1, \quad \text{if} \quad h \geqslant 5,$$

.....

$$n_r = 4 \cdot 5^{r-1} + 1, \quad \text{if} \quad h \geqslant r.$$

It is easily verified by taking $x = (\ldots 00003)$ that the least value of n_2, valid for *all* x of the specified type, is 21. Hence it follows that the values of n_3, n_4, ... given above are the least values. Since $\{N_r\} - 1$ is a factor of $\{\lambda N_r\} - 1$, the most general value of n_r is $4\lambda \cdot 5^{r-1} + 1$, if $h \geqslant r$.

But for all values of x, prime to 10, h is at least 4. Hence the problem reduces to the determination of n_r when $r > 4$ and $4 \leqslant h < r$. We now deal with this case.

3.2. Writing $n_r' = 4 \cdot 5^{r-1} + 1 = N_r' + 1$, then

$$\{n_r'\} - x \equiv 0 \pmod{2^h \cdot 5^r}.$$

3.21. We assume that

$$\{N'_{r-1}\} - 1 \equiv 0 \pmod{2^h \cdot 5^{r-1}}.$$

Then $$\sum_{q=1}^{4} (\{kN'_{r-1}\} - 1) \equiv 0 \pmod{2^h \cdot 5^{r-1}},$$

that is,

3.22. $$1 + \{N'_{r-1}\} + \{2N'_{r-1}\} + \{3N'_{r-1}\} + \{4N'_{r-1}\} \equiv 0 \pmod 5.$$

Multiplying 3.21 by 3.22,

$$\{5N'_{r-1}\} - 1 \equiv 0 \pmod{2^h \cdot 5^r}.$$

Thus it is sufficient to take $N_r' = 5N'_{r-1} = 4 \cdot 5^{r-1}$;

that is, $$n_r' = 4 \cdot 5^{r-1} + 1.$$

The left-hand side of 3.22 is odd, since it is the sum of an odd number of odd numbers. Hence the power of 2 contained in $\{N_r'\} - 1$ will be the same as that in $\{N'_{r-1}\} - 1$, and hence by reduction the same as that in $x^4 - 1$. Hence the result.

3.3. $$n_r = 4k \cdot 5^{r-1} + 1 = N_r + 1, \text{ where } k = 2^{r-h}.$$

Since by 3.2, $$\{N'_r\} - 1 \equiv 0 \pmod{2^h 5^r},$$

we shall have $$\{N_r\} - 1 \equiv \{kN'_r\} - 1 \equiv 0 \pmod{10^r}$$

if $$(\{N'_r\} - 1)[\{(k-1)N'_r\} + \ldots + \{N'_r\} + 1] \equiv 0 \pmod{10^r},$$

that is, if $$\{(k-1)N'_r\} + \ldots + \{N'_r\} + 1 \equiv 0 \pmod{2^{r-h}}.$$

Since the left-hand side consists of k terms, all odd, we must have $k = 2^{r-h}$.

Then $N_r = k \,.\, N'_r$, that is,

$$n_r = 4k \,.\, 5^{r-1} + 1.$$

3.4. This value of n_r is the least of the numbers for which

$$x^{n_r} - x \equiv 0 \pmod{10^r}$$

for *all* x, as is evident by considering $x = (\ldots\ 00\ \ldots\ 03)$.

S. PARAMESWARAN.

1896. *A useful graph.*

1. The multi-purpose graph described below may be of interest. The writer uses it :

(*a*) to introduce logarithms intuitively,
(*b*) to illustrate the later formal theory of indices and logarithms, and
(*c*) to introduce the exponential function e^x in calculus.

2. The graph is $y = (4/3)^x$. It is plotted for integral values of x from -7 to $+8$ inclusive ; y varies from 0 to 10 approximately and, in fact, $(4/3)^8$ is almost exactly 10. The graph fits conveniently into the usual 8″ by 13″ graph sheet, taking $\frac{1}{2}$″ as unit on the x-axis and 1″ as unit on the y-axis. The positive powers are computed in succession by adding to each power one-third of itself ; the negative powers are obtained from tables of reciprocals.

3. For purpose (*a*), one first revises the positive integral index shorthand and the rules for $a^m \times a^n$ and $a^m \div a^n$.

3.1. Assuming one of these rules, meanings are found for negative integers and zero as indices.

The other rule is shown to hold with these meanings.

Fractional indices are not mentioned at this stage.

3.2. The graph described in 2. is plotted.

3.3. The operations of multiplication, division and then of finding positive integral powers and roots (the former regarded as repeated multiplication and the latter by reversing the powers procedure) are performed intuitively on the graph. The axes are re-labelled " Numbers " and " Logarithms ".

3.4. Logarithms to base 10 and logarithm tables are introduced.

4. Under heading (*b*), one may find logarithms to base 10 by dividing logarithms read from the graph by 8 [since $(4/3)^8 = 10$], and the results may be checked from the tables.

5. For purpose (*c*), one first tabulates ordinates and gradients from the graph for $x = 0, 1, \ldots, 7$, (accuracy being difficult to attain on the negative side).

One finds that for $y = (4/3)^x$, $dy/dx = ky$ where $k = \cdot 28$ or $\cdot 29$.

5.1. By the function of a function rule in calculus, one deduces that if $y = (4/3)^{mx}$, then $dy/dx = mky$.

5.2. Hence, $dy/dx = y$ if and only if $mk = 1$; *i.e.* if and only if

$$y = (4/3)^{x/k},\ dy/dx = y.$$

5.3. Write $e = (4/3)^{1/k}$.

From reciprocal tables and the graph, $e \doteqdot 2{\cdot}7$.

5.4. Since $(4/3)^{x/k} = \{(4/3)^{1/k}\}^x = e^x$, it follows from 5.2. that if $y = e^x$, $dy/dx = y$ and e is the only number with this property.

5.5. The usual treatment of e^x and $\log_e x$ follows. As a final check, one may calculate $\dfrac{d}{dx}\left[\log_{4/3} x\right]$ and compare with gradients read off from the graph.

P. GANT.

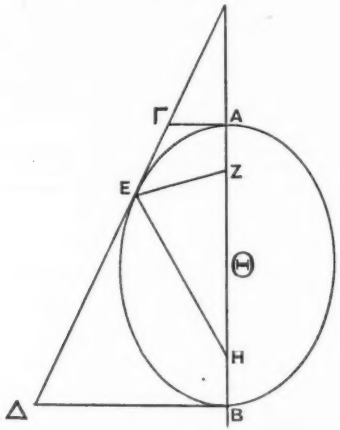
1897. *Why S and H?*

In a book recently advertised in the *Gazette* we are asked: Why are the letters S and H often used for the foci of an ellipse? The author's own answer is not given, and before committing ourselves, in the "astronomical" context, to some rash suggestion such as Sol and Helios, it may be useful to consider the following facts:

(1) The letters are "often used" in this country, but hardly abroad.

(2) NEWTON uses them (*Principia*, 1687).

(3) APOLLONIUS gives, of course, only very few focal properties, viz. III, 45–52, where the diagrams are of the type illustrated; clearly the notation simply follows the Greek alphabet, Z, H are the "next" letters in their natural order.



(4) The *ed. princeps* of the Greek text of Apollonius appeared at Oxford in 1710; the editor was ED. HALLEY (who had also "edited" the *Principia* for Newton). The MSS. from which the ed. was compiled are in Bodley's Library (Sav. 59, Sav. 10, Sav. 7 and others); they contain just these letters, either actually in diagrams or in the text when the diagrams are missing; the MSS. are not "old", but of the sixteenth and seventeenth centuries.

(5) Latin editions of Apollonius (*Princeps*, 1566, ed. COMMANDINO) follow the Latin alphabet, and so have E, F, or f, g for the foci. BARROW's ed. of 1675 adheres to this usage.

(6) In the Latin edition of PAPPUS (1602, ed. COMMANDINO) the letter C is used at the place where the focus-directrix property is given.

(7) Other seventeenth-century authors on Conics—or Optics—vary in their usage of letters, and even of the name: KEPLER says *focus*, and uses no special letters; NEWTON speaks of *umbilicus*; GREGORIUS A. S. VINCENTIO of *polus*; the Greeks, *i.e.* APOLLONIUS and PAPPUS, "had no word for it".

A. P.

1898. *A sun-dial formula.*

This Note gives a formula for approximating to the difference between time registered by a sun-dial and Greenwich Mean Time. This difference is divided into two parts. The first, the "equation of time", due to the eccentricity,

is measured by the difference between the hour angles of the true and dynamical mean Suns. The second, due to the obliquity, is measured by the difference between the dynamical and astronomical mean Suns.

The former can be written as

$$\tan^{-1}\tfrac{1}{30}\sin(\theta - 12°),$$

and the latter as

$$\tfrac{299}{300}[{}^*\sin^{-1}\sin\theta - \sin^{-1}\{\sin\theta\,.\,(1-\sin^2 i\cos^2\theta)^{-\frac{1}{2}}\}],$$

where $\theta = 0$ at the winter solstice, 22nd December,

and $\theta = 360°$ at the winter solstice on the following year.

The expression ${}^*\sin^{-1}\sin\theta$ must be interpreted as follows:

$$\sin^{-1}\sin\tfrac{1}{2}\pi = \tfrac{1}{2}\pi,$$
$$\sin^{-1}\sin\pi = 0, \text{ etc.}$$

We solve in degrees and then multiply by 4 to convert into minutes. The multiplication by 299/300 decreases the total error.

To complete our task it is essential to convert degrees into number of days. This can be done by the formula that the number of days is approximately

$$\tfrac{487}{480}\{(\theta - \cdot 4) - 1\cdot 91\sin(\theta - 12°)\},$$

where θ is in degrees.

Further, if h is the hour angle, l the latitude of the place, and ∂ the declination of the sun, so that

$$\sin\partial = \sin i\cos\theta;$$

then

$$\cos h = -\tan l\tan\partial - \cdot 0148\sec l\sec\partial.$$

This, combined with the "equation of time" formula, given above, gives times of sunrise and sunset. R. H. BIRCH.

1899. *The angle-theorem of the circle*: $AP_1B \equiv AP_2B$ ("Complete Angle" congruence).

Returning to this subject after many years,* the simplest proof of the general theorem would appear to be as follows:

A, B being given points of a circle (centre O) and P the representative point of it, then if Q, R be the mid-points of the arcs AP, BP, respectively (and A, Q, P, R, B in that order on the circle), the lines OQ, OR are perpendicular to PA, PB respectively; hence the (ordered) line-pairs (PA, PB) and (OQ, OR) are congruent. Also,

$$\text{arc-length } QPR = \tfrac{1}{2}\,.\,\text{arc-length } APB,$$

and is, therefore, constant for variation of P on *either* of the arcs AB (and thence, also, $\angle QOR$ is constant—in sign as well as magnitude). Hence, for such variation of P, all the line-pairs (PA, PB) are congruent with one another.

In the limiting positions A and B of P, the line-pair (OQ, OR) takes the positions (OA, p) and (p, OB), respectively, where p denotes the diameter which is perpendicular to AB. And these belong to *both* of the arcs AB.

Hence for *all* positions of P on the circle,

$$(PA, PB) \equiv (OQ, OR) \equiv (OA, p) \equiv (p, OB)$$

and $$\equiv (\alpha, AB) \equiv (AB, \beta),$$

if α, β denote the tangents at A, B respectively. D. K. PICKEN.

* See *Math. Gaz.*, XI, 161 (Dec. 1922), p. 190, and *Proc. London Math. Soc.*, Ser. 2, XXIII, p. 55. Other parts of the *Gazette* article are relevant. There is a copy of the other paper ("Euclidean Geometry of Angle") in the M.A. Library.

CORRESPONDENCE.

EXPONENTIAL AND LOGARITHMIC FUNCTIONS.

To the Editor of the *Mathematical Gazette*.

SIR,—Mr. Tuckey's Note 1805 (*Mathematical Gazette*, February, 1945, p. 23) is to the point; I would, however, suggest that in this case a double entry table is more useful than the graphical representation. We require the proof of three things:

(i) that a^x is continuous, in other words, $a^h \to 1$ as $h \to 0$;

(ii) that $(a^h - 1)/h$ tends to a limit which depends on a;

(iii) that when this limit is 1, a is about 2·71828.

I have found the following sufficient.

Table I gives values of a^h for $a = 1, 2, 3, 10, 100, 1000, 1000000$, and for $h = 0{\cdot}1, 0{\cdot}01, 0{\cdot}001, 0{\cdot}0001$. The values should be obtained by seven-figure logarithms and the convergence to 1 is shown clearly even when a is one million.

Tables II giving values of $(a^h - 1)/h$ for the above values of a and h can then be written down at sight, and shows the convergence to a limit increasing with a; for example, 0·69 when $a = 2$, and 1·10 when $a = 3$. This shows that e will lie between 2 and 3.

The derivatives of e^x and $\log x$ follow, and we can calculate $\log_e 2$ from the integral $\int_1^2 dx/x$ by Simpson's rule to 6 places, and having $\log_{10} 2$ from the tables we get $\log_{10} e$ easily to 5 places and so e itself. Yours, etc.,

Melbourne. R. J. A. BARNARD.

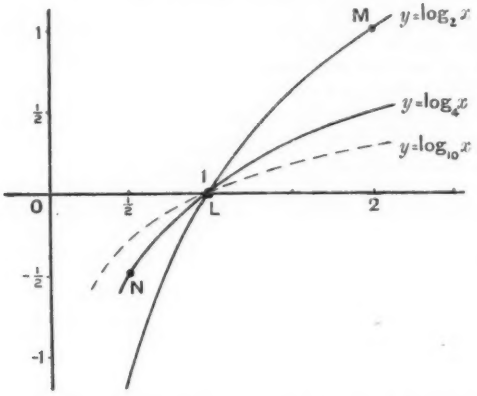
To the Editor of the *Mathematical Gazette*.

SIR,—Most teachers will agree with Messrs. Durell and Robson that the method of introducing the logarithm by investigating hyp $(t) \equiv \int_1^t dx/x$ is exciting, has outlook value and should be employed with mathematical specialists. On the other hand, it is difficult. It involves transforming a definite integral, concluding that if hyp $(e^n) = n$, then hyp $(x) \equiv \log_e x$, *i.e.* that there is only one function f which has the property $f(e^x) = x$, and finally proving a special case of

$$\frac{d}{dt}\int_c^t g(x)\ dx = g(t).$$

Although it is more natural to think of integrating $1/x$ than of differentiating a^x or $\log_a x$, the latter process is much easier, and need not really be much of a "bolt from the blue". After investigating algebraic and trigonometric functions and finding that the latter are sometimes required for integrating the former, it is quite natural to consider the logarithmic function as another function met with in elementary work. Nearly every boy, as Mr. Tuckey has pointed out, will know in advance that $D \log_e x = 1/x$. If he hasn't looked ahead in the textbook he will have seen it at the end of his tables, on the back of his slide-rule or in his diary. For the incurious innocent the calculation of $\log(a + h)$ in terms of $\log a$ and h or the fact that the gradient of $y = \log x$ appears to be inversely proportional to x could be made starting-points for the investigation.

I prefer to differentiate $\log_a x$ rather than a^x, using the following method, for which no originality is claimed. Plot $y = \log_a x$ for $a = 10$, 2 and 4. Since $\log_a x = \log_a b \times \log_b x$, the curves $y = \log_a x$ for variable a are such that any one of them is a sideways stretch from the x-axis of any other.



On the curve $y = \log_a x$ consider two points P, Q whose abscissae are x_1 and $x_1(1+h)$. The gradient of the line PQ is

$$\frac{1}{x_1} \cdot \frac{\log_a (1+h)}{h}.$$

The gradient of the tangent at P is μ_a/x_1, where

$$\mu_a = \lim_{h \to 0} \frac{\log_a (1+h)}{h}.$$

Thus (i) the gradient of $y = \log_a x$ is proportional to $1/x$;

(ii) μ_a is the gradient at the point $L(1, 0)$ on the curve and μ_a decreases steadily as a increases.

Consider now in particular the curves $y = \log_2 x$ on which the point $M(2, 1)$ lies and $y = \log_4 x$ on which the point $N(\frac{1}{2}, -\frac{1}{2})$ lies. Since the gradient of these curves decreases as x increases they are concave downwards and

$$\mu_2 > \text{gradient of } LM = 1;$$

$$\mu_4 < \text{gradient of } LN = 1.$$

Thus there is a number e between 2 and 4 for which $\mu_e = 1$, and the gradient of $y = \log_e x$ is $1/x$. Yours, etc.,

R. C. LYNESS.

TECHNICAL MATHEMATICS.

To the Editor of the *Mathematical Gazette*.

SIR,—As a mathematics teacher who has just returned to school after four and a half years doing research work in industry and helping with part-time technical classes, I was very interested to read the report in the October

Gazette of the discussion at the General Meeting on Technical Mathematics. I would like to support the suggestion that the M.A. approach the Engineers' Institutions with a view to drawing up a syllabus. From my experience, unlike Dr. Booth's, there is a growing awareness in industry of the need for mathematics, especially among the younger engineers. During the war it has been very difficult to find suitably-trained people for industrial posts from the lowest to the highest, and both the Ministry and some firms have shown a willingness to consider temporary methods of filling this gap. With peace this need is not going to decrease if all branches of British industry are to reach the top scale of technical efficiency. With more time and the changes in education due to the new Act we will be able to tackle the problem more effectively. I would suggest that perhaps the discussion could be continued at the next General Meeting of the Association, that joint meetings of the local branches of the Association, the Engineering Institutes, and other interested bodies could be arranged to discuss the common problems, and that some arrangements could be made to report the discussions in the technical engineering press.

It is clear that the discussions taking place in Britain can be paralleled by those in other countries. Several speakers mentioned America, where there was considerable disquiet about the fact that most inter-war mathematical research was on the pure side, and that very little had been done on the applied side. Amongst other things Brown University arranged several Vacation courses on Mechanics, and helped to sponsor the new *Quarterly of Applied Mathematics* with a distinguished editorial board. In this country we lack a suitable journal of applied mathematics. I think the M.A. would be performing a very useful service if it obtained articles relating these and other experiences which would help us here. There are several pointers, too, that the experience of the U.S.S.R. would be helpful; they are known to have placed more emphasis on applied mathematics, but still, unfortunately, co-operation with this country leaves a lot to be desired.

It would be a great mistake if grammar school teachers were to assume that this discussion about technical mathematics did not concern them. Several speakers mentioned the possibility of a common course for grammar and technical sides of secondary schools, whether or not they were part of the same school. It seems obvious that many more grammar-school-trained students will find employment in industry in the future, and we will have to consider what repercussions this will have on our syllabus, even if we are not teaching in the experimental multilateral schools. If I may give a few examples, I think we should consider whether we have gone far enough with our changes in the geometry syllabus, and whether there are other branches of mathematics more worthy of inclusion for both educational and vocational reasons. Every course of logarithms should include some remarks about the slide rule as a very commonly-used means of calculating and, where possible, children should be shown the simple adding machine. In my experience I hardly used any of the geometry I had learnt, and did all the calculation I needed either on a slide rule or a calculating machine and hardly ever opened a book of logarithm tables.

Another grammar-school problem of some importance for all except the largest and most liberally staffed is that of giving adequate time and attention to sixth form work—I was distinctly amused to read of the discussion on sixth form work, where it was assumed that teachers had about twice as many periods for the sixth form as I am allowed. It is obvious that time does not press only in technical part-time courses. Under the new Act, however, all pupils will remain at school until sixteen and then have two years at the County College. What is going to happen to the sixth forms of grammar

schools where they maintain an independent existence and are not part of a comprehensive school? It seems to me that there are educational as well as social reasons for combining sixth form and intermediate pupils as much as possible so as to allow a better staffing ratio. If I may quote my own experience, we have at the moment four grammar schools and a technical college all trying to provide an adequate H.S.C. and Intermediate course. Then, in addition, are the evening courses at the technical college. I strongly support the plea made by several speakers that technical training should be given during the day. Already more private firms are making this possible, and with the nationalisation of other industries the government could make this practice the rule. I do not agree with the suggestion made by one speaker that day courses should be vocational, but think that the cultural side should be treated as well, with further facilities being provided in the evenings for the more enthusiastic. In case any reader supposes that I am against cultural education, I would here like to urge that as a profession we mathematics teachers must do much more to make our subject really cultural, but to deal with this adequately would take a whole article.

Mention was made of the possibility of industrial personnel being able to bring mathematical problems to the staff of technical colleges for assistance. While I support this whole-heartedly, as one to whom such problems might be brought, I would also like to urge the organisation of a consultative service on a wider sphere, because very often the industrial problems raise mathematical difficulties in which only the really expert specialist can help. If there are much wider facilities for post-graduate research, there will be many more engineers with the necessary mathematical background and mathematicians with the engineering knowledge to make co-operation between the two sides much more fruitful.

Several speakers mentioned details of possible syllabuses. I would like to point out that very few mentioned what is normally called mechanics, and this agrees with my experience that what is required is a more fundamental knowledge of pure mathematics and, first of all, of functions of a complex variable. Many of us found that our most-used book was Whittaker and Watson's *Modern Analysis*, although there is obviously room for a book covering approximately the same ground and designed for the engineer. It may interest others to know that we used elliptic functions very extensively (although naturally not as much as Bessel functions), particularly in connection with Schwarz transformations to provide solutions of the Laplace equation. This example also brings home the lack of suitable textbooks for the mathematical engineer, who is hardly at all concerned with the Weierstrass $\wp$ function and only with the theta functions to the extent that they are necessary to deal with the elliptic integral of the third kind, which is only just mentioned in Whittaker and Watson.

After complex variables I would place numerical methods as very necessary and important. A real understanding of these methods is important even when a closed mathematical solution is possible, as the answer required is generally numerical. Usually, however, an exact solution is not possible, and some estimation of the error made is important. Often different methods have to be combined, such as using series expansions about two different points to estimate the value of the function at an intermediate point, or in finding an approximate solution of Laplace's equation by using bits of exact, numerical and graphical (curvilinear squares) solutions of allied problems. In connection with machines, it must be realised that it is insufficient to know how to use a machine to perform ordinary operations, but it is desirable to know and develop methods for use on particular machines, quite apart from the large research problem of developing new machines and adapting existing

machines to solve new problems. Other topics I would like to mention include determinants and matrices, vectors, and Fourier series and integrals.

In conclusion, Sir, may I wish the Mathematical Association all good luck in its efforts to improve the understanding of technical mathematics at all levels. Yours sincerely,

P. J. WALLIS.

AN OLD PROBLEM.

To the Editor of the *Mathematical Gazette*.

SIR,—I have been interested lately by the old unsolved problem of placing n points in m straight lines of p in a line, so that m is a maximum. I should be grateful if any members could inform me of the literature on the subject: I believe both Newton and Sylvester studied it.

It might interest some of your readers to try their hands at the following selection of possibilities, to each of which I have at least one solution. The numbers are the values of p, n, and m, in that order. 3, 11, 16; 3, 12, 19; 4, 18, 18; 4, 24, 28; 4, 25, 30; 4, 36, 55; 5, 22, 15; 5, 26, 21; 5, 30, 26.

The question originally proposed was to find n_0, the minimum value of n for which $m \geqslant n$ for any value of p. When $p=3$, $n_0=9$, and when $p=4$, $n_0=18$, so it is tempting to guess that when $p=5$, $n_0=36$ and, in general, $n_0=9\cdot 2^{p-3}$, but I have not so far found such a solution even for $p=5$.

Yours, R. H. MACMILLAN.

SIXTH FORM SYLLABUSES

To the Editor of the *Mathematical Gazette*.

SIR,—There is temerity in commenting at a distance on the discussion about Sixth Form syllabuses recorded in the *Mathematical Gazette* for October, especially as the clearly valuable report on the training of physicists is not available here, nor are the Cambridge Entrance Scholarship papers which are so highly praised. But our experience for a score of years with Descriptive Mathematics appears to be relevant, and some attempt may be made to state its significance for the changing situation in England.

The discussion concerned two subjects which are well and clearly distinguished, the mathematical preparation best for various future courses of study, and that for winning scholarships with a view to the study of mathematics. Only to the former can I refer, though I note no reference to the consequences of concentration on scholarship work for those who fail to get a scholarship, and though I gather from the constitution of the Joint Advisory Committee the impression that University representation is rather strong. About this I feel that teachers should have clearer views as to aims in teaching which are not too directly linked to the study of advanced mathematics; University teachers can keep in touch with such aims, and help to determine them, but for this they need not have great voting strength on committees.

First, I note too little emphasis, notwithstanding the prominence given to Statistics and the mention of actuarial mathematics, on the uses of mathematics in connection with the social sciences. Not forgetting Marshall's warnings, it may be said that these uses are specially urgent now in regard to Economics. (An example of fumbling may be easily seen at p. 285 of *Employment, Interest and Money*.*) We need a committee or group of

*More positively, there is the apparent fact that *Econometrica* flourishes in the U.S.A., and it is doubtful if it is appreciated as generally in England.

teachers of mathematics to search the writings on the social sciences and see in how great a degree incisive study of these sciences is lacking owing to unfamiliarity with certain types of what might be school mathematics. A group is needed, too, to deal similarly with Biology, and probably another group for the exact sciences, though these almost monopolised attention in the discussion on the need to make school mathematics more applicable. These are large tasks, but there is no escaping them if among the educated in days to come there is to be a critical and a constructive attitude in any degree adequate to the complexities of the times.

The above may seem to lack in consideration for the cultivation of advanced mathematics, and so I hasten to add that we have found the contrast that is common between pure and applied mathematics rather unreal. It may be that I use "applied" here in a wide sense, but this fits in with the demand from the universities, as well as from girls' schools, for more unification. I should like, too, to see our experience confirmed that such unification does not conflict with the possibility of dealing within a comparatively simple framework with a large variety of topics rather than with a few thoroughly, which is a desideratum for technical mathematics. It seems to me that it is desirable in the immediate future to make a more searching scrutiny of the details of mathematics that suit the types of pupils which have been already so well demarcated—though there is a type that needs, and does not know that it needs, mathematics of a certain kind. Of such may have come those scholars who protest now against mathematical over-elaboration, while the real difficulty is that mathematics has not been used with discrimination. For this we teachers may be responsible because we have not looked around enough.

JOHN MACLEAN.

1488. Les théories générales, pour prendre dans la Science un droit de cité définitif, ont le plus souvent besoin de s'illustrer par des applications particulières. Dans plusieurs domaines, celles-ci ne sont pas toujours faciles à trouver, et l'on pourrait citer, dans les Mathématiques modernes, plus d'une théorie confinée, si j'ose le dire, dans sa trop grande généralité.—C. E. Picard, in his introduction to the *Œuvres de Galois*, 1897. [Per Prof. E. H. Neville.]

1489. *Detective Logic.* If I may venture a suggestion, you appear to attach too much importance to the actual inmates of the house. Your line of reasoning is that they had the best opportunity of entering the study. That is certainly true. But do not forget that others had an equal opportunity.—John Rhode, *Poison for One*, 1934, p. 130. [Per Professor E. H. Neville.]

1490. If he borrowed from Sir Isaac Newton the sublime method of fluxions, Leibnitz was at least the Prometheus who imparted to mankind the sacred fire which he had stolen from the gods.—Gibbon, "Antiquities of the House of Brunswick," Ch. 1, Sect. 1 (*Misc. Works*, 1814 ed., III, 362). [Per Professor E. H. Neville.]

FOR SALE.

A. A. Albert: *Modern Higher Algebra*. Pp. 318. Cambridge University Press, 1938. Published at 18s. One copy only, *as new*, offered at 12s. 6d.

Offers to K. E. Reading, 3 Grendon Gardens, Wembley Park, Middlesex.

REVIEWS.

The psychology of invention in the mathematical field. By J. HADAMARD. Pp. xiii, 143. 13s. 6d. 1945. (Princeton University Press; Humphrey Milford)

This is, apart from one famous lecture of Poincaré *, the first attempt by a mathematician of the first rank to give a picture of his own modes of thought and those of other mathematicians. Professor Hadamard has earned the thanks of all of us for his courage in undertaking such a task, formidable from both its psychological and its mathematical difficulties, and there is one thing at any rate which he has certainly established, that the most important of the qualifications required is to be a good mathematician. He can sometimes quote illuminating remarks from " outsiders ", philosophers, poets, or even literary critics, but nothing appears more clearly from the book than the comparative futility of the speculations of psychologists who have not been mathematicians. Galton alone, of the outside authorities quoted, shows up really well; a few pages from Poincaré or Hadamard about theorems they have found themselves have ten times the value of the theories of Gall, Möbius †, Nicolle, Paulhan, Souriau, and the rest of them. One might add that " inductive " enquiries by questionnaire, such as that conducted by *L'enseignement mathématique* in 1902, have on the whole proved equally unilluminating.

We must therefore all be grateful to Professor Hadamard for this stimulating little volume, written with all the authority of one of the greatest mathematicians of the last fifty years, and with the most charming frankness concerning his own achievements, triumphs and failures alike. Indeed it is the most personal parts of the book which seem to me the most attractive. A lesser mathematician would never have explained to us how, by some odd mischance, he failed to discover " Jensen's theorem ", or how, having found the " Lorentz group " of transformations, he dismissed it as " obviously devoid of physical meaning ". A lesser mathematician may perhaps also derive some comfort from the recital of such misadventures.

I must begin, however, with two criticisms which I hope may not seem ungracious. The book is too condensed—I regret myself that it is not at least fifty per cent. longer. And I regret still more that it is not written in French: after all, there would soon have been an English translation. It is true that Professor Hadamard's English is (as anyone who has talked with him, or heard him lecture, can testify) far better than that of most Frenchmen, most even of those who have lived for years in an English-speaking country. But to write for publication to so general an audience, on a subject so packed with subtleties as this, is a test quite different from conversation, or lecturing, or the composition of a technical memoir. The result is that, while the meaning is almost always clear, and the language generally " correct ", it lacks the idiomatic force of Poincaré's French, and the whole book, especially in its earlier and more definitely psychological chapters, gives a certain impression of crabbedness and congestion. Writing in French, Professor Hadamard would naturally have been more expansive.

In this respect I find a considerable difference between the earlier and later chapters. The first part of the book is dominated by discussions of the " unconscious " and its role in discovery, an obscure subject which might daunt even an experienced psychologist, and here Professor Hadamard is wisely diffident and tentative in his conclusions. The contents of the later chapters are more miscellaneous, and include much more that is both unquestionable and easy to follow.

* Ch. 3 ('L'invention mathématique') of *Science et méthode*.

† The neurologist, not the mathematician.

I must begin by saying something about the earlier and more difficult chapters. The main facts enumerated in them seem beyond dispute. That unconscious activity often plays a decisive part in discovery; that periods of ineffective effort are often followed, after intervals of rest or distraction, by moments of sudden illumination; that these flashes of inspiration are explicable only as the result of activities of which the agent has been unaware—the evidence for all this seems overwhelming. Poincaré's is the classical example, but a quite ordinary mathematician can recognise similar experiences. How these unconscious activities are related to those of a more normal kind, to fully conscious work or reflection on the fringe of consciousness, how they function and what is the proper language in which to describe them, are terribly difficult questions. But all of us can remember problems, even on the examination level, whose solution has come to us suddenly when we have all but forgotten them, and it is hard to believe that the rest given by forgetfulness is a sufficient explanation.

Poincaré's experiences (in the discovery of the "theta-fuchsian" functions and the transformations which govern them) were exceptional in some ways, but most of us can remember experiences similar in kind; we have only to recall the best work we have done as the result of prolonged and strenuous effort. The typical course of events is something as follows. There is a first stage of fully deliberate activity, possibly with some, but certainly with unsatisfactory results—Poincaré's were entirely negative. Then a rest, complete or partial, compulsory or deliberate, the result of other occupation or diversion to different problems, followed by a moment of sudden illumination. Then a second period of conscious effort, this time successful, in which the broad outlines of the solution become clear. Then, very likely after long delay, the final stage of what Hadamard calls "precision", in which the results are "written up" and put in order, a tiresome and subsidiary but essential process. These four periods seem the minimum, but of course there may be more; Poincaré's experiences were a good deal more elaborate, and he had at least two moments of unexpected inspiration.

The mystery lies entirely in the early stages, and first in the initial stage of "preparation", which Hadamard considers in detail in Ch. 4. It is plain that during this stage, however futile it may have seemed, we have done something essential; we have shaken up our ideas in a way which somehow makes later illumination possible. To "discover" is to combine ideas fruitfully, and we have at any rate formed a mass of combinations. These have seemed useless, and most of them have been really uninteresting; but the process of forming them has been less unfruitful than it has seemed, since it has set in motion the unconscious machinery which would never have operated without such an initial disturbance. It is a most difficult process to describe, and neither Poincaré nor Hadamard does more than suggest vague images; but the vaguest image of a mathematician may be more profitable than the theory of a psychologist who has never made a mathematical discovery in his life, and Poincaré has one of which Hadamard seems to approve. He compares the ideas which are the future elements of our combinations to "the hooked atoms of Epicurus". While our mind was inactive, these atoms were motionless; they were "hooked to the wall", but our efforts set them in motion. "After this shaking up imposed upon them by our will, they do not return to their primitive rest, but continue their dance freely", and have impacts with one another resulting in fresh combinations. In the new combinations thus formed, indirect results of the original conscious work, lie the possibilities of apparently spontaneous inspiration.

So far this is plausible enough, but we have still to *select* from among all these combinations: "invention is discernment, choice", and where and

how is this choice made ? This is the most puzzling question, and I cannot feel that either Poincaré or Hadamard points at all clearly to any satisfactory reply. It seems plain that our unconscious activities must have included some process of selection, since most of our unconscious combinations never rise to our consciousness at all. Poincaré and Hadamard find the solution in the unconscious working of our aesthetic sense. " The privileged unconscious phenomena, those capable of becoming conscious, are those which affect our emotional sensibility most profoundly. . . . In what region of the mind does this sorting take place ? Surely not in consciousness, which, among all possible combinations, knows only the right ones. . . . To the unconscious belongs the most delicate and essential task, that of selecting those which satisfy our sense of beauty and are therefore most likely to be fruitful. . . ." It may be so, though I cannot say that I find it very convincing ; but I am no psychologist, and my distaste for all forms of mysticism may be prejudicing me unduly.

It is admitted that there are more commonplace explanations of some at any rate of the phenomena. Our mind, when we return to the problem, is freshened by its rest ; we have escaped from " interferences which block progress during the stage of preparation " ; we have " got rid of false leads and hampering assumptions, and can approach the problem with a more open mind " ; and it is quite natural that we should be more successful. But I am forced to agree with Hadamard that " while such explanations can be admitted in some cases, in others they are contradicted by the facts " : they do not account for experiences like that of Poincaré when he boarded the omnibus at Coutances. We seem driven to admit that " the unconscious is not merely automatic, it has tact and delicacy ", even that " it knows better how to divine than the conscious self, since it succeeds where that has failed ". But I do not *like* this kind of language, and if, with Poincaré, we begin asking " is not therefore the subliminal self superior to the conscious self ? ", then I have an uncomfortable feeling that we are rather near talking nonsense (and I gather from a remark on p. 42 that Hadamard does not altogether disagree with me).

It is something of a relief to pass to the later chapters, which are full of interesting and less controversial matter. In particular Ch. 6 (" Discovery as a synthesis ") is a long and important one. It contains a detailed discussion, with many examples, of the use of *signs*, both by mathematicians generally and by Hadamard himself ; and here I find his conclusions, though they seem to be well substantiated, rather astonishing. He may be justified in his contempt for Max Müller's view that thought is conducted almost entirely by *words*, but he goes to the other extreme ; it seems that his mind, like Galton's, is of just the opposite type. " I insist that words are entirely absent from my mind when I really think " (and here " words " are to be understood in a very wide sense, including, for example, algebraical symbols). " I use them only when dealing with easy calculations ; whenever the matter becomes difficult, they become too heavy a baggage for me. I use concrete representations, but of a quite different nature. . . . Words remain absolutely absent from my mind until I come to the moment of written or oral communication." It is an extreme view, stated with surprising emphasis, but it is supported by definite instances ; Hadamard explains, for example, his " picture " of the proof of the infinitude of the primes. And, although I should have expected his case to be exceptional, he has found, as a result of quite extensive enquiries, that most mathematicians agree with him. There were only two among those whom he consulted who confessed to belonging to the " verbal " type, Birkhoff, who " visualized algebraical symbols ", and Pólya, who seems dominated by words even more than I am myself. For my own

part I must confess that I think almost entirely in words and formulae, written if possible, or visualised as printed on paper, and that for this reason I find thought almost impossible if my hands are cold and I cannot write in comfort. If Hadamard thinks with his legs (" except in the night when I cannot sleep, I never find anything except by pacing up and down the room "), then I think with my fingers. It may be a humiliating confession, but surely I have one illustrious companion in sin—did not Euler say, somewhere, that he thought with his pen ?

Another arresting discussion is that, in Ch. 7, of " different types of mathematical minds ", with special reference to the familiar contrast between the " intuitive " and " logical " types. The subject is of course one about which a great deal of nonsense has been talked. Even in 1893 we find Klein declaring that " a strong native space intuition seems to be an attribute of the Teutonic race, while the critical and purely logical sense is more developed in the Latin and Hebrew races "—and we have all heard this doctrine later in much cruder forms. It is pleasant to find that Hadamard preserves his sense of humour, and quotes French dicta of equal absurdity.

Hadamard recalls Poincaré's comparison of two pairs of famous mathematicians. Poincaré, like Hadamard, rises above the nationalistic prejudice which stakes out a claim to the gifts which sound most impressive, and chooses two pairs of mathematicians of the same nationality, Bertrand (perhaps a little overshadowed in such company) and Hermite, Riemann and Weierstrass. And here Hadamard reveals a very interesting difference of opinion. Bertrand, he agrees, " had visibly a concrete and spatial view of every question ", but it seems to him absurd to class Hermite as a " logician ", " Nothing can appear to me more directly contrary to the truth. . . . Methods always seemed to be born in his mind in some quite mysterious way. . . . I can hardly imagine a more perfect type of an intuitive mind than Hermite's." He agrees that Hermite " was not used to thinking in a concrete way " and " had a positive hatred for geometry ", and concludes that the association, so often suggested, of physics and geometry with intuition, analysis with logic, may sometimes have very little foundation. I think that one might also quote Ramanujan, an " intuitive " mathematician, surely, if ever there was one, for the same purpose.

About Riemann and Weierstrass, Hadamard agrees better with Poincaré; and no doubt this antithesis is generally accepted and corresponds broadly to the facts. Yet even here one must be cautious about accepting such distinctions too readily. I do not know whether Hadamard has studied Siegel's analysis of Riemann's *Nachlass* on the Zeta-function. Riemann, we were told, was the outstanding example of a mathematician dominated by broad and general ideas, and here he reveals himself as a formalist quite of Ramanujan's type. It is enough to shake anybody's confidence in these facile generalisations.

Here Hadamard suggests two criteria of his own, based on his earlier discussions of the " unconscious " (while warning us that we must not expect to find them always concordant). First, we might reasonably describe a mind as " intuitive " if its original combinations of ideas are formed in a comparatively deep layer of the unconscious, as " logical " if this layer is comparatively superficial. Secondly we might (reverting to Poincaré's image of the " hooked atoms ") define the intuitive mind as one in which the initial disturbances of the atoms are notably random and scattered, the logical mind as one in which, even from the beginning, they follow comparatively narrow and convergent paths. The criteria seem plausible enough, but it is significant that, in the particular case which Hadamard considers most closely, they lead to contradictory results. Galois, he finds, was a highly " intuitive " mathe-

matician according to the first criterion, a highly "logical" one according to the second.

I have no space to refer to more of Hadamard's many arresting discussions—the whole book is packed with provocative matter. But I cannot refrain from one final expression of satisfaction. In his last chapter he gives a short discussion of the motives which inspire research, and I am naturally delighted to find that his views agree substantially with my own. Indeed he states the case more strongly than I should have dared to do myself. That little first-rate mathematics is done with a view to immediate application; that "it seldom happens that mathematical researches are undertaken directly in view of a given practical aim"; that "practical questions are most often solved by means of existing theories", and that the applications, however important, are usually remote in time; that, in short, research is normally inspired by "the common motive of all scientific work, the desire to know and to understand"—all this is surely (whatever the ardent young politicians of the Archimedeans may say) common ground among ninety per cent. of mathematicians, and the experiences of an Hadamard or a Hilbert merely confirm those of their humbler colleagues.

G. H. HARDY.

Currents in Aerials and High-Frequency Networks. By F. B. PIDDUCK. Pp. vi, 97. 8s. 6d. 1946. (Oxford)

This book deals with currents in aerials for radio reception and transmission, and their associated networks. The frequency is assumed to be high enough for the current to be confined mainly to a thin layer at the surface of the conductor of circular cross-section. The treatment is based upon a short paper by H. C. Pocklington in *Proc. Camb. Phil. Soc.*, 1897. The fundamental formulae are deduced from this paper, and were first given by F. H. Murray in *Amer. Journ. Math.*, 1931. The treatment is more rigorous than that usually found in texts dealing with the same class of problem. Some of the topics treated are as follows: equal parallel aerials near a resonant length; current in a straight receiving aerial; aerial arrays; Lecher wires; feeders. The analysis depends to a large extent on the exponential integral

$$\mathrm{Ei}(\pm ix) = \mathrm{Ci}(x) \pm i\,\mathrm{Si}(x).$$

Tabular values of $\mathrm{Ci}(x)$, $\mathrm{Si}(x)$ are given from $x=0$ to $x=10$ with interval 0·01, and from $x=10$ to $x=29{\cdot}9$ with interval 0·1. There are also tables of $\cos x$ and $\sin x$ from $x=0$ to $x=10$ with interval 0·01.

In order to treat aerials parallel to the ground, the function

$$E^s{}_n(x) = C^s{}_n(x) + i\,S^s{}_n(x) = \int_0^\pi e^{inv} J_0(xy)\, y^s\, dy$$

is introduced. When x is large enough, $E^s{}_n(x)$ is expressible in terms of Lommel functions. Tabular values of $C^0{}_1$, $S^0{}_1$, and their first derivatives, also $C^1{}_1$, $S^1{}_1$ are given from $x=0$ to $x=2$ with interval 0·05 and from $x=2$ to $x=10$ with interval 0·2.

The analytical part of the text will be of interest to applied mathematicians. It is unfortunate, however, that many of the expressions are of great length, sometimes exceeding half a page. This detracts from the reader's concentration on the problem at issue. On the whole the text is not easy to follow, and at times the descriptive matter lacks precision. For instance, on p. 10: "The resistance of the aerial is in general swamped by radiation from the aerial. . . ." What the author means is that, at very high frequencies, the radiation resistance of an aerial is much greater than its ohmic resistance.

Or, alternatively, the power radiated from the aerial is much greater than that dissipated as heat in the aerial.

Since the accuracy claimed for the method used is greater than that obtained by standard procedure, a numerical comparison of the results obtained by both methods would have enabled the reader to judge their relative merits. The acid test is, of course, comparison with practical measurements, provided they can be made with accuracy adequate for this purpose.

If the text were expanded and made easier to follow, and the use of the complicated expressions in the analysis justified by practical measurements, this book would be a valuable contribution to the literature of the subject.

N. W. McL.

Weltfahrt der Erkenntnis. Leben und Werk Isaac Newtons. By F. DESSAUER. Pp. 432, with 8 full-page plates. Cloth. 17·50 francs. 1945. (Rascher Verlag, Zürich)

The first half of the title, "Weltfahrt der Erkenntnis" (lit. "World-conquest of Knowledge") gives the clue and purpose of this book. Dr. Dessauer takes us with him on the adventurous voyage of discovery upon which young Newton embarked. He wishes to bring the reader into contact with the substance of the discoveries as they were in Newton's mind, and with the changing mode of thought which links Newton's work with Galileo's. We are enabled to keep vigil with Newton during the long nights of intensive thought and share his dawn thrill of seeing for the first time in all their clarity the laws which govern the universe; of fathoming the secrets of the planetary orbits, of light, of the new mathematics and the laws of force.

The author is at pains to prove that Newton was a child of his age, but also that he was a prisoner of his own *ego*. He paints him as a naturally reserved character whose soul was completely ruled by an excessively vigorous thought-life; a hypersensitive man, distrustful of and hostile to all external influences; a fundamentally hard-living man who hedged in his interior life to preserve its fecundity.

The reader who accompanies this (perhaps the greatest) English genius on his journey through life will be rewarded by a new understanding of our own times; by the realisation that the fate of man is determined by mind; and by a keener appreciation of the fact that the foundations of the future are already laid in the present.

Dr. Dessauer shows himself here, as in his works on Galileo and Röntgen, to be a master in this field of writing. It is a book that should be read by all students of physics; and will be enjoyed by all who are interested in the history and development of thought.

There is a rich store of information in the copious notes and supplements.

J. B.

Physics and Experience. By BERTRAND RUSSELL. Pp. 26. 1s. 6d. 1946. The Henry Sidgwick Lecture, 1945. (Cambridge)

This lecture exhibits all the invigorating charm that we expect to find in anything Lord Russell has to say. To read it is somewhat like taking a brisk walk over the hills: one feels toned up by such an excursion, even if one does find oneself at the end of it at the same place as one started from. Russell, naturally, tries to lead us to a different place. He has a lot to say about "percepts", and it may well be that one's own percepts are inadequate to detect the difference.

The question which he sets out to discuss is: "Assuming physics to be broadly speaking true, can we know it to be true, and, if the answer is to be in the affirmative, does this involve knowledge of other truths besides those

of physics? " * He is of the opinion that this has been far too little considered. However, to an amateur in such matters, most of his actual discussion seems to deal with topics with which philosophers have always concerned themselves, as, for instance, the relation between " the sun that I see " and " the astronomer's sun ". Further, the amateur cannot avoid the impression that there lurk, somewhere in the background, some of those traditional preconceptions which really *make* the problems under discussion. One's trouble is that one is far from clear as to what Russell means here by " physics ". Consider, for example, his statement, " While it is necessary to distinguish between perceptual space and physical space, it is not necessary to make an analogous distinction as regards time." Thinking in terms of the four-dimensional manifold of events of general relativity, this appears as a most unnatural assertion : why should one dimension be distinguished from the rest by a relation to perception? On the other hand, in Milne's kinematical relativity, physical time is *by definition* perceptual time, and almost the same may be said in regard to Robb's theory of time and space. So, in the absence of further explanation as to what Russell himself means by physical time, it is difficult to attach a precise meaning to his statement. But the latter has been quoted in order to suggest also a further difficulty. Whether or not one accepts the conclusions of kinematical relativity, its development does serve to show that the passage from perceptual time to *measurable* time is a matter of exceedingly deep argumentation, and not until it has been carried through can we legitimately claim to have something that may be called physical time. Physics deals only with what can be measured. But neither in regard to time nor anything else does Russell refer explicitly to measurement. On perfectly general grounds one would not expect to get very far in a discussion of *Physics and Experience* which gives no prominence to the metrical aspect of physics.

To the mind of the reviewer, the questions here discussed would seem most likely to be solved by the very different sort of approach to physics which was sketched out by Eddington. One dare even suggest that the solution would in some cases be reached by showing the questions to be largely meaningless. But for one not possessing Russell's superb dialectic skill, any attempt to discuss these matters more fully would occupy much more than the mere two dozen pages of the lecture itself. W. H. McC.

Introduction to Non-Euclidean Geometry. By HAROLD E. WOLFE. Pp. xiii, 247. $4.50. (The Dryden Press, New York)

The dust-cover claims that this is " the First Textbook on Non-Euclidean Geometry designed for classroom use ". Though this may be an overstatement (the scope is very similar to that of Carslaw, for example), the book is certainly a carefully prepared and well-written account of the subject, and may be recommended to all who are interested.

In the preface the author stresses the importance of a study of Non-Euclidean Geometry to all who would fully understand the Euclidean : " Advanced study in Euclidean Geometry is not the only requisite for the good teaching of Euclid. The study of Non-Euclidean Geometry takes its place beside it as an indispensable part of the training of a well-prepared teacher of high school geometry." One feels, in fact, that this may be the chief use of the book in this country, where it is unlikely to slip into any of the syllabuses at present under consideration. Many schools might, however, find it a welcome addition to their library.

The first three chapters are introductory. In the first, the foundations of Euclidean Geometry are passed in review. The second deals in detail with Euclid's fifth postulate and with attempts to prove it, and the third is a

historical sketch of the growth of the subject. With chapter four, we begin a detailed account of hyperbolic plane geometry, using the Saccheri quadrilateral, the Lambert quadrilateral, and so on. The fifth and sixth chapters, still on hyperbolic geometry, consider trigonometry and the use of the calculus respectively. Chapter seven contains a similar treatment of elliptic plane geometry and trigonometry. The final chapter is devoted to an examination of the consistency of the Non-Euclidean geometries, dealing, in particular, with the analogous problem of the circles orthogonal to a given circle. In an appendix we find, usefully, a list of the postulates and propositions of Euclid, Book I, and, less usefully perhaps, a brief account of the circular and hyperbolic functions, orthogonal circles, and inversion. There are also a number of exercises throughout the book, though I doubt whether that number will be considered to deserve its description " adequate ", at any rate for teaching purposes.

On reading the book with the subject-matter not very fresh in my mind, I found that, though the account was usually very clear, there was one point which gave me trouble. When he begins the study of Hyperbolic Geometry, the author naturally does not want to enumerate all the Euclidean theorems which remain valid ; his space is too short. He is content to remark : " All of the Euclidean propositions which do not depend on the Fifth Postulate, in particular the first twenty-eight, are immediately available." This, to me, implies that Euclid's whole argument up to that point remains sound under the new hypothesis. But surely the method of superposition which Euclid used (however reluctantly) must now be rejected, and the reader is left wondering whether the congruence theorems go with it. Actually the author has already given Hilbert's Postulate System in the first chapter, on *Euclidean* geometry, but he does not make it clear, as far as I can see, that the congruence postulate for two triangles is still to be retained for the hyperbolic plane. I felt, as I read it, that more explanation should have been given at this point.

Once the first notions have been grasped, the reader will, I think, find here a good account of the subject. The printing and general presentation of the book seem excellent.

E. A. MAXWELL.

Cours d'Analyse mathématique. Par G. VALIRON. *Théorie des fonctions.* Pp. ii, 522. 300 fr. 1942. *Équations fonctionnelles ; applications.* Pp. ii, 605. 600 fr. 1945. (Masson, Paris)

We hope to be able to call attention in these columns to some of the books published in Europe during the war which, for that reason, could not be reviewed at the date of publication.

MM. Masson have very courteously supplied a copy of Professor Valiron's *Cours d'Analyse*, (1942, 1945), a treatise in the classical tradition by an author who is well known for his researches, particularly on the theory of integral functions. It is not practicable to discuss in detail the contents of the 1100 pages of the two volumes, especially as these are very closely packed by a type of small fount, so that the ground covered is extremely extensive. In spite of the small type, careful setting and a judicious use of display prevent the pages from appearing overcrowded.

The first volume deals with the theory of functions of a real and of a complex variable. The preliminaries are rather briefly sketched, and are inadequate if regarded as a self-contained exposition : but of course under the French system the reader would have already covered much of this early ground. Professor Valiron says in his preface that his aim is two-fold : first, to prove and extend those general theorems which in the junior courses have been taken for granted without proof ; secondly, to develop new theories and techniques. Thus the first volume deals mainly with the definite integral,

multiple integrals, and Cauchy's form of the theory of functions of a complex variable. This is all familiar ground ; the exposition is lucid, finished, but not forward-looking. The Lebesgue integral, for example, is given a chapter, but this has almost the appearance of an appendix, and the central and fundamental position of this integral in modern analysis is not at all emphasised. When dealing with the complex variable, the development leads more rapidly to present-day features, and gives the work a more vital interest.

The second volume has a somewhat unusual title, which is intended to describe the main topics, namely the theory of algebraic functions, Riemann surfaces and Abelian integrals, and the theory of ordinary and partial differential equations. Among the applications there is a chapter on the calculus of variations, and four chapters on the differential geometry of curves and surfaces. The work on differential equations occupies more than half the volume, and is therefore equivalent to a fairly substantial treatise in itself. For ordinary differential equations the emphasis is laid firmly, and properly, on existence theorems in the real and complex domains ; but there are very few relevant matters which are not included. For example, numerical methods of solution are given a brief treatment, and as an example from a widely different angle, there is also something about Lie's theory. On partial differential equations, one chapter deals with first order equations, existence theorems and the methods of Cauchy, Lagrange, Mayer, Charpit and Jacobi ; the second deals with second order equations, existence theorems, the elliptic, hyperbolic and parabolic types of equation, and some applications.

A new *Cours d'Analyse* inevitably challenges comparison with the classic treatises familiar to every self-respecting student of mathematical analysis. The reviewer has always felt that of the four best-known classics, Goursat and de la Vallée Poussin exhibit a perfection of completeness and finish, while Jordan and Picard display the beauty and stimulus of a living and active organism. Such a contrast, if accurate, does not disparage either side ; and if we say that Valiron's book has more of the air of Goursat and de la Vallée Poussin, the implied mild regret is surely more than outweighed by the magnitude of the compliment. If, at times, the brilliant and lucid exposition of these authors leaves us feeling that nothing remains to be done, the brilliancy and lucidity still stand as well worth emulation. T. A. A. B.

Mathematical Tables. I. Prepared by the British Association Committee for the calculation of Mathematical Tables. 2nd edition. Pp. xi, 72. 10s. 1946. (Cambridge University Press)

The splendid series of British Association mathematical tables is universally recognised as a most valuable production, and great praise is due to the Committee ; in this place we may perhaps not inappropriately give particular mention to the large share taken in the Committee's labours by our own Librarian, Professor E. H. Neville.

Volume I now appears in a second edition, the most important change being the omission of the greater part of the Introduction, wherein the origin, nature and interpolation of the tables was fully described. Enough remains for the utilitarian purchaser ; bibliographers must still consult the first edition. It may be added that the price of the new edition is not increased.

A list of the tables, argument interval and number of decimal places will sufficiently indicate the contents :

multiples of $\frac{1}{2}\pi$ [1(1)100 ; 15] ;

circular sines and cosines [radians : 0·0(0·1)50·0 ; 15 : 0·000(0·001)1·600 ; 11] ;

hyperbolic sines and cosines of πx [0·0000(0·0001)0·0100 ; 15 : 0·00(0·01)4·00 ; 15] ;

hyperbolic sines and cosines of x [0·0(0·1)10·0 ; 15]
exponential integral [0·0(0·1)15·0 ; 11] ;
sine and cosine integrals [0·0(0·1)5·0, 5·0(0·1)20·0(0·2)40·0 ; 11, 10] ;
factorial function [0·00(0·01)1·00 ; 12] ;
logarithmic factorial integral [0·00(0·01)1·00 ; 10] ;
digamma, trigamma, tetragamma, pentagamma functions *
[0·00(0·01)1·00, 10·0(0·1)60·0 ; mainly 12] ;
two tables for the Hermitian probability functions. T. A. A. B.

* Namely $D^n \log_e(x!)$, for $n = 1, 2, 3, 4$.

BOOKS RECEIVED FOR REVIEW.

A. Fletcher, J. C. P. Miller and L. Rosenhead. *An index of mathematical tables.* Pp. viii, 451. 75s. 1946. (Scientific Computing Service)

A. Geary, H. V. Lowry and H. A. Hayden. *Advanced mathematics for technical students.* I. Pp. viii, 419. 12s. 6d. 1946. (Longmans)

C. Godfrey and A. W. Siddons. *The teaching of elementary mathematics.* 2nd edition. Pp. xi, 322. 7s. 6d. 1946. (Cambridge)

A. Grosrey. *Éléments de Calcul Infinitésimal.* Pp. 192. 280 fr. 1945. (Gauthier-Villars)

J. E. Kerrich. *An experimental introduction to the theory of probability.* Pp. 98. Dan. Cr. 8.50. 1946. (Munksgaard, Copenhagen)

A. Mercier. *Leçons et problèmes sur la théorie des corps déformables.* Pp. 156. 240 fr. N. d. (Gauthier-Villars)

F. S. Nowlan. *Analytic geometry.* 3rd edition. Pp. xiv, 355. $2.75. 1946. (McGraw-Hill)

W. J. Stone. *Building Geometry.* Pp. 213. 7s. 6d. 1946. Building Craft Series (Longmans)

W. P. Workman. *School arithmetic.* 3rd edition, revised by G. H. R. Newth. Pp. viii, 548. 5s. 6d. ; without answers, 5s. 1946. (University Tutorial Press)

The gyroscope and its applications. Edited by MARTIN DAVIDSON. Pp. 256. 21s. 1946. (Hutchinson)

Mathematical Cuneiform Texts. Edited by O. NEUGEBAUER and A. SACHS. Pp. x, 177, 49 pp. plates. 5 dollars. 1945. American Oriental Series, 29. (American Oriental Society, New Haven, Conn.)

Mathematical Tables. I. Circular and hyperbolic functions, exponential and sine and cosine integrals, factorial function and allied functions, Hermitian probability functions. Prepared by the British Association Committee for the Calculation of Mathematical Tables. 2nd edition. Pp. xi, 72. 10s. 1946. (Cambridge)

Scientific instruments. Edited by H. J. COOPER. Pp. vii, 293. 25s. 1946. (Hutchinson)

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